# GLOBAL POSITIONING SYSTEM STANDARD POSITIONING SERVICE SIGNAL SPECIFICATION

#### ANNEX A

# STANDARD POSITIONING SERVICE PERFORMANCE SPECIFICATION



2nd Edition

June 2, 1995

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#### **SECTION 1.0 SPS Minimum Performance Standards**

This Annex specifies the minimum performance that an SPS user can expect to experience, when equipped with an SPS receiver designed and operated in accordance with the SPS Signal Specification. Performance is specified in terms of minimum performance standards for each performance parameter. Each standard includes a definition of conditions and constraints applicable to the provision of the specified service. SPS performance parameters associated with the standards are defined in Section 1.4.2 of the SPS Signal Specification. See Annex B for a more detailed discussion of each performance parameter, and a description of expected SPS performance characteristics. See Annex C for specific information regarding the measurement of performance against each standard.

Any performance parameters not specified in this Annex are not considered to be part of the minimum SPS performance standards, or to represent a part of the minimum service being provided to the civil community.

In the standard definitions below, two terms are used that require clarification: *global average* and worst-case point. The definition of a standard in terms of a global average represents a conservative average performance which a user located at any arbitrary location on or near the Earth can expect to experience. The definition of a standard in terms of a worst-case point represents a bound on the performance which a user located at the worst possible location on or near the Earth can expect to experience.

Note that accuracy performance standards are based upon signal-in-space error characteristics and their effects on the position solution. The standards do not include the contribution of the SPS receiver to range or position domain error.

## **SECTION 2.0 Coverage Standard**

SPS coverage will be provided in accordance with the following tolerances.

Coverage Standard	Conditions and Constraints
≥ 99.9% global average	<ul> <li>Probability of 4 or more satellites in view over any 24 hour interval, averaged over the globe</li> <li>4 satellites must provide PDOP of 6 or less</li> <li>5° mask angle with no obscura</li> <li>Standard is predicated on 24 operational satellites, as the constellation is defined in the almanac</li> </ul>
≥ 96.9% at worst-case point	<ul> <li>Probability of 4 or more satellites in view over any 24 hour interval, for the worst-case point on the globe</li> <li>4 satellites must provide PDOP of 6 or less</li> <li>5° mask angle with no obscura</li> <li>Standard is predicated on 24 operational satellites, as the constellation is defined in the almanac</li> </ul>

## **SECTION 3.0 Service Availability Standard**

SPS service availability will be provided in accordance with the following tolerances.

Service Availability Standard	Conditions and Constraints
≥ 99.85% global average	<ul> <li>Conditioned on coverage standard</li> <li>Standard based on a typical 24 hour interval, averaged over the globe</li> <li>Typical 24 hour interval defined using averaging period of 30 days</li> </ul>
≥ 99.16% single point average	<ul> <li>Conditioned on coverage standard</li> <li>Standard based on a typical 24 hour interval, for the worst-case point on the globe</li> <li>Typical 24 hour interval defined using averaging period of 30 days</li> </ul>
≥ 95.87% global average on worst-case day	<ul> <li>Conditioned on coverage standard</li> <li>Standard represents a worst-case 24 hour interval, averaged over the globe</li> </ul>
≥ 83.92% at worst-case point on worst-case day	<ul> <li>Conditioned on coverage standard</li> <li>Standard based on a worst-case 24 hour interval, for the worst-case point on the globe</li> </ul>

## **SECTION 4.0 Service Reliability Standard**

SPS service reliability will be provided in accordance with the following tolerances.

Service Reliability Standard	Conditions and Constraints
≥ 99.97% global average	<ul> <li>Conditioned on coverage and service availability standards</li> <li>500 meter NTE predictable horizontal error reliability threshold</li> <li>Standard based on a measurement interval of one year; average of daily values over the globe</li> <li>Standard predicated on a maximum of 18 hours of major service failure behavior over the sample interval</li> </ul>
≥ 99.79% single point average	<ul> <li>Conditioned on coverage and service availability standards</li> <li>500 meter Not-to-Exceed (NTE) predictable horizontal error reliability threshold</li> <li>Standard based on a measurement interval of one year; average of daily values from the worst-case point on the globe</li> <li>Standard based on a maximum of 18 hours of major service failure behavior over the sample interval</li> </ul>

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## **SECTION 5.0 Positioning and Timing Accuracy Standard**

GPS positioning and timing accuracy will be provided in accordance with the following tolerances.

Accuracy Standard	Conditions and Constraints
Predictable Accuracy ≤ 100 meters horizontal error 95% of time ≤ 156 meters vertical error 95% of time ≤ 300 meters horizontal error 99.99% of time ≤ 500 meters vertical error 99.99% of time	<ul> <li>Conditioned on coverage, service availability and service reliability standards</li> <li>Standard based on a measurement interval of 24 hours, for any point on the globe</li> </ul>
Repeatable Accuracy ≤ 141 meters horizontal error 95% of time ≤ 221 meters vertical error 95% of time	<ul> <li>Conditioned on coverage, service availability and service reliability standards</li> <li>Standard based on a measurement interval of 24 hours, for any point on the globe</li> </ul>
Relative Accuracy ≤ 1.0 meters horizontal error 95% of time ≤ 1.5 meters vertical error 95% of time	<ul> <li>Conditioned on coverage, service availability and service reliability standards</li> <li>Standard based on a measurement interval of 24 hours, for any point on the globe</li> <li>Standard presumes that the receivers base their position solutions on the same satellites, with position solutions computed at approximately the same time</li> </ul>
Time Transfer Accuracy ≤ 340 nanoseconds time transfer error 95% of time	<ul> <li>Conditioned on coverage, service availability and service reliability standards</li> <li>Standard based upon SPS receiver time as computed using the output of the position solution</li> <li>Standard based on a measurement interval of 24 hours, for any point on the globe</li> <li>Standard is defined with respect to Universal Coordinated Time, as it is maintained by the United States Naval Observatory</li> </ul>
Range Domain Accuracy ≤ 150 meters NTE range error ≤ 2 meters/second NTE range rate error ≤ 8 millimeters/second² range acceleration error 95% of time ≤ 19 millimeters/second² NTE range acceleration error	<ul> <li>Conditioned on satellite indicating healthy status</li> <li>Standard based on a measurement interval of 24 hours, for any point on the globe</li> <li>Standard restricted to range domain errors allocated to space/control segments</li> <li>Standards are not constellation values each satellite is required to meet the standards</li> <li>Assessment requires minimum of four hours of data over the 24 hour period for a satellite in order to evaluate that satellite against the standard</li> </ul>

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# GLOBAL POSITIONING SYSTEM STANDARD POSITIONING SERVICE SIGNAL SPECIFICATION

#### ANNEX B

# STANDARD POSITIONING SERVICE PERFORMANCE CHARACTERISTICS



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#### **SECTION 1.0 Introduction**

GPS performance behavior is dynamic, paticularly when it is compared with systems such as LORAN-C. The dynamic nature of GPS performance is understandable, given the four-dimensional nature of the position solution and the use of satellites as mobile beacons. GPS performance may however be defined in a straightforward fashion, and bounds or standards placed upon the range of performance a user will experience. These bounds are established in the Annex A as SPS performance standards. The proper context in which to viewGPS performance standards is provided through a definition of expected variations for each aspect of system performance.

#### 1.1 Purpose

This Annex defines expected GPS SPS performance parameters and their characteristics, as a function of time, user location, system design and changing operational conditions. This Annex defines the civil GPS performance envelope associated with the minimum performance standards established in Annex A.

#### 1.2 Scope

The data contained in this Annex provides a context for proper understanding and interpretation of civil minimum performance standards established in the GPS SPS Signal Specification. The data and associated statements provided in this Annex represent conservative performance ex pectations, based upon past system performance. Note that this Annex contains material which is illustrative of GPS performance characteristics, and should not be considered to be definitive. Future GPS performance will not necessarily be, but is expected to be consistent with the characteristics described in this Annex.

The GPS SPS Signal Specification establishes new definitions and relationships between traditional performance parameters such as coverage, service availability, service reliability and accuracy. GPS performance specifications have previously been made to conform to definitions which apply to fixed terrestrial positioning systems. The new definitions are tailored to better represent the performance attributes of a space-based positioning system.

#### 1.3 An Overview of SPS Performance Parameters

System behavior is defined in terms of a series of performance parameters. These parameters are statistical in nature, to better represent performance variations over time. The four performance parameters dealt with in this Annex are: coverage, service availability, service reliability and accuracy, as shown in Figure 1-1. The characteristics of each of these parameters must be considered to completely define the GPS civil performance envelope.

A very important relationship exists between these performance parameters. Performance definition begins with coverage. Each successive layer of performance definitions are conditioned on the preceding layers. For example, coverage must be provided before the service may be con

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sidered available, it must be available before it can support service reliability requirements, and the service must be performing reliably before accuracy standards may be applied.

#### 1.3.1 Coverage

GPS coverage is viewed somewhat differently than coverage for terrestrial provided positioning systems. Traditionally, coverage has been viewed as the surface area or volume in which a system may be operated. Since a terrestrial system's beacons are fixed, coverage does not change as a function of time. Since the GPS concept relies upon the dynamics of a satellite constellation, coverage must take into consideration a time dependency. GPS coverage is by definition intended to be global. GPS coverage is viewed alternatively as the percentage of time over a time interval that a user, any where in the world and at any time, can see a sufficient number of satellites to generate a position solution. Constraints are placed upon satellite visibility in terms of mask angle and geometry, to minimize the possibility of a SPS receiver generating a marginal position solution. Coverage characteristics over any given region vary slightly over time, due primarily to small shifts in satellite orbits.

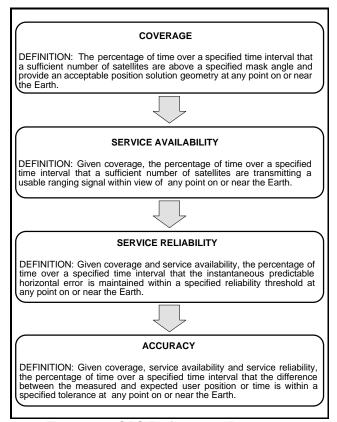


Figure 1-1. GPS Performance Parameters

#### 1.3.2 Service Availability

Just because a satellite is operational does not mean that it is currently transmitting a usable SPS ranging signal. Satellites will, on occasion, be removed temporarily from service for routine maintenance. As a result, the number of satellites actually transmitting usable ranging signals will vary over time. **Service availability** is the measure of how GPS coverage deviates from nominal conditions due to the temporary removal of satellites from service. This measurement represents the percentage of time that coverage is provided by those satellites which are transnitting usable ranging signals to generate a position solution. Variations in service availability are a function of which satellites are removed from service, the length of the service outage, and where on the globe a user is located in relation to any resulting outage patterns.

#### 1.3.3 Service Reliability

GPS can be used anywhere in the world. A failure in a system with such global coverage may affect a large percentage of the globe. A natural concern about using GPS is whether or not it provides a satisfactory level of **service reliability**. Service reliability as it is used in a GPS context is somewhat more restrictive than the classical definition, which includes times that the service is available as well as when it is performing within specified tolerances. GPS service reliability is viewed as a measure only of how well GPS maintains horizontal errors within a specified reliability error threshold. 100% service reliability is provided when the horizontal error does not exceed the reliability error threshold, within the conditions specified for coverage and service availability.

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Periods where the service does not provide a sufficient number of satellites or adequate geometry to support position solution generation are assessed against the coverage or service availablity performance standards.

GPS service reliability is a function of several factors. The primary factors are the failure fequency, and duration of the SPS ranging signal service failure. Once a ranging signal service failure has occurred, the probability that a user at any arbitrary location will experience a reliability failure due to the service failure depends on:

- The user's location relative to the failed satellite's coverage pattern,
- The amount of time that the failed satellite is in view if the user is within some portion of the coverage pattern,
- The probability that the user will use the failed satellite in the position solution, and
- The probability that the magnitude of the failure will be large enough to induce a service reliability failure, based upon the specific solution geometry through which the error is leing mapped.

#### 1.3.4 Accuracy

Given that coverage is provided, the service is available and all satellites are performing within reliability tolerances, GPS position solution *accuracy* represents how consistently the receiver's output conforms to an expected solution. Users view accuracy in many different ways, depending on their application. To accommodate the majority of users' needs, GPS positioning accuracy is defined in the Signal Specification from four different perspectives:

- Predictable Accuracy,
- Repeatable Accuracy,
- Relative Accuracy, and
- Time Transfer Accuracy.

Each of these aspects of GPS accuracy are described in more detail below. Figure 1-2 compares and contrasts the four different ways of viewing GPS accuracy as it is defined in the Signal Spedication.

**Predictable accuracy** represents how well the position solution conforms to "truth". Truth is de fined to be any specified user location where the position is surveyed with respect to an accepted coordinate system, such as the World Geodetic System 1984 (WGS-84) Earth-Centered, Earth-Fixed (ECEF) Coordinate System. GPS was implemented to specifications that are stated in terms of predictable accuracy. Predictable accuracy is a measure used by those who are concerned with how well they can position themselves relative to a known, surveyed location. Factors which affect predictable accuracy include geometry variations unique to a given user location, and the sample interval over which measurements are taken.

**Repeatable accuracy** is a measure of position solution consistency relative to a user's previous position solution. Users who are interested in returning to points where they previously used GPS to determine their position will rely upon GPS repeatable accuracy performance. Repeatable accuracy varies primarily as a function of time between measurements.

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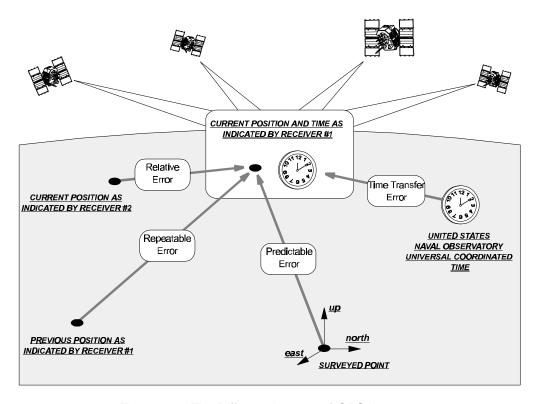


Figure 1-2. The Different Aspects of GPS Accuracy

Relative accuracy is a measure of the correlation in the errors between position solutions from two different receivers, using the same satellites at approximately the same time. Users who wish to locate other receivers relative to their location are most concerned with relative accuracy. Ideally, only very small differences will exist between the position solutions of two receivers that are relatively close together and consistently use the same satellites. These differences will be due primarily to receiver designs and measurement noise plus the difference in solution generation times. Other factors which can potentially contribute relative solution errors are slight differences in solution geometries and ranging errors between the two sites. These factors provide a negligible contribution to relative errors, as long as the receivers are within 40 kilometers of each other. The 40 kilometer constraint is based simply on the fact that it becomes increasingly difficult beyond that distance to base the two position solutions on common-view satellites that provide a position solution geometry within Position Dilution of Precision (PDOP) constraints.

**Time transfer accuracy** defines how well a position service user can relate receiver time to Universal Coordinated Time (UTC) as it is disseminated by the United States Naval Observatory (USNO).

#### 1.4 Key Terms and Definitions

The terms and definitions of technical concepts provided below should be reviewed to ensure a common understanding of the material presented in this Annex.

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**Measurement Samples.** A group of measured quantities that meet random sampling criteria when they are taken from a specified population. Each of the measured quantities are explicitly grouped by a specified measurement process and by the measurement interval over which the measurements were taken.

**Measurement Interval.** The time interval over which measurement samples are gathered from a specified population to evaluate an aspect of system performance.

**Stationarity.** A measure of statistical behavior consistency over successive sample intervals for a specified sample population. Individual satellite ranging errors which provide consistent mean and variance statistics over successive sample intervals may be viewed as being sufficiently sationary. This specific view of stationarity is also known as *wide-sense stationarity*.

**Ergodicity.** The degree to which the statistical behavior of instantaneous samples from several populations conform to the statistical behavior of samples from one population over a sample interval. A series of satellites with similar and stationary ranging error statistics over successive sample intervals may be viewed as behaving in an approximately ergodic manner.

Steady-State. Behavior within statistical expectations.

**Transient.** Short term behavior not consistent with steady-state expectations.

**Position Solution Geometry.** The set of direction cosines which define the instantaneous reationship of each satellite's ranging signal vector to each of the position solution coordinate axes.

**Dilution of Precision (DOP).** A Root Mean Square (RMS) measure of the effects that any given position solution geometry has on position errors. Geometry effects may be assessed in the local horizontal (HDOP), local vertical (VDOP), three-dimensional position (PDOP), or time (TDOP) for example.

**User Navigation Error (UNE).** Given a sufficiently stationary and ergodic satellite constellation ranging error behavior over a minimum number of measurement intervals, multiplication of the DOP and a constellation ranging error standard deviation value will yield an approximation of the RMS position error. This RMS approximation is known as the UNE (UHNE for horizontal, UVNE for vertical, and so on). The user is cautioned that any divergence away from the stationary and ergodic assumption will cause the UNE to diverge from a measured RMS value.

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## **SECTION 2.0 Coverage Characteristics**

This section defines GPS constellation design objectives, and the characteristics of GPS coverage which are expected with a 24 satellite operational constellation. The user is provided with general information concerning how coverage will vary over time on a global basis, and a worst-case projection of coverage on a regional basis. The data provided in the discussion is based upon a global assessment of grid points spaced equally, approximately 111 kilometers apart, every 30 seconds over a 24 hour period.

#### 2.1 The GPS 24 Satellite Constellation

The 24 satellite constellation is designed to optimize global coverage over a wide range of opeational conditions. Specific constellation design objectives are listed below:

- 1) Provide continuous global coverage with specified geometry and mask angle constraints.
- 2) Minimize coverage sensitivity to expected satellite orbital drift characteristics.
- 3) Mitigate the effects on service availability of removing any one satellite from service.

Several factors affect GPS coverage. These factors must be taken into consideration in the constellation design. The factors are:

- The difference between the planned orbit and the orbit actually achieved during the launch and orbit insertion process,
- Orbit variation dynamics, and
- Frequency and efficiency of satellite stationkeeping maneuvers.

### 2.2 Expected Coverage Characteristics

Proper support of Design Objective 1 from above requires that at least four satellites are continuously in view with an acceptable geometry and mask angle anywhere in the world. An implication of this requirement is that most of the time significantly more than four satellites will be visible. As shown in Figure 2-1, eight satellites will be visible on average for any location in the world, over 24 hours. Very seldom will a user see only four satellites when all 24 satellites are providing usable ranging signals. If the 24 satellites in the GPS constellation were all launched with no deviations into their planned orbits, and no drift were allowed, the constellation would provide virtually 100% (0.99999714) four satellite coverage with a PDOP constraint of 6.

Unfortunately, variations in final orbits based upon launch uncertainties and routine drift do occur. Design Objective 2 is supported by evaluating how changes in each satellite's orbital elements affect nominal coverage characteristics. Bounds are applied to orbital element deviations from the nominal orbit to ensure that constellation coverage does not degrade beyond allowed limits. Degraded coverage areas drift and change slightly in shape over time, but their average number and duration will remain approximately constant for a given constellation. Changes in the number of satellites or significant shifts in satellite orbits however can dramatically change the attributes of degraded coverage areas.

Given a 24 satellite constellation, GPS will provide 100% four and five satellite coverage without a PDOP constraint (but with a mask angle of 5), and six satellite coverage greater than 99.9% of the time. However, four satellite coverage with a PDOP constraint of 6 can drop as low as 99.9%, with a worst-case dispersion of the 24 satellites with respect to their nominal orbits. Even in this event, most users will experience continuous coverage. A few isolated locations may experience four-satellite coverage as low as 96.9%, with a PDOP constraint of 6 and a mask angle of 5°.

Satisfaction of Design Objective 3 requires that we be able to remove any individual satellite from the constellation, and still be able to provide as close to continuous global coverage as is practical. Satisfaction of this objective requires that at least five satellites be in view almost continuously. As shown in Figure 2-1, this is the case with the 24 satellite constellation design. Although an explicit requirement is not established to ensure that multiple combinations of satellites provide adequate solution geometry at any given time, most of the time at least two and usually more combinations of four satellites will support a Position Dilution of Precision (PDOP) constraint of 6 or less.

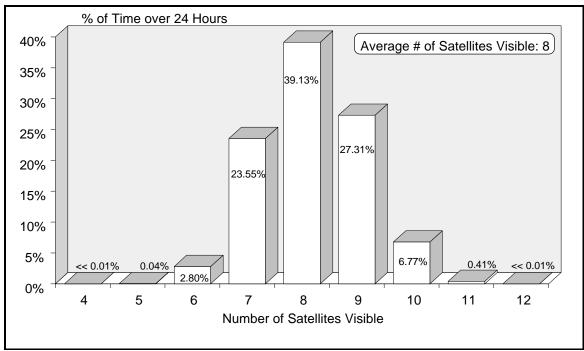


Figure 2-1. Satellite Global Visibility Profile

A final point on coverage performance relates to the term "on or near the Earth" used throughout the Signal Specification. Since GPS is a space-based system, coverage is defined as a function of each satellite's navigation signal beamwidth. The GPS satellite's nominal beamwidth is approximately  $\pm 14.3^{\circ}$ . If a user on the Earth's surface were to view a satellite which is just above the local horizon, the user could elevate from that location to an altitude of approximately 200 kibmeters above the Earth's surface before effectively losing that satellite's signal. This condition defines the maximum altitude associated with the term "on or near the Earth".

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## **SECTION 3.0 Service Availability Characteristics**

This section defines expected regional and global service availability characteristics. The user is provided with information concerning GPS service availability patterns on a global and regional basis. Service availability varies slightly over time, due to routine satellite maintenance requiements. Note that the regional service availability values provided below are based upon a global grid point spacing of approximately 111 x 111 kilometers, with 30 second intervals over 24 hours.

Service availability is described in two basic parts. The first part concerns the variation in service availability as a function of temporarily removing a number and specific combination of satellites from service. The second part of the assessment applies service availability variation characteristics to an operational scenario.

#### 3.1 Satellite Outage Effects on Service Availability

Service availability varies predominantly as a function of the number and distribution of satellite service outages. With a 24 satellite constellation, the permutations and combinations of satellite service outages are rather large. Normally, no more than three satellites will be removed from service over any 24 hour interval. This groundrule bounds the problem to an analysis of the effects of removing each satellite and all combinations of two and three satellites from service for no more than 24 hours. The results of the analysis are summarized in Table 3-1.

Table 3-1. Service Availability as a Function of Specified Satellite Outage Conditions

Satellite Temporary Outage Condition	Global Average Service Availability	Worst Regional Service Availability	
No Satellites Out:	100%	100%	
One Satellite Out for Maintenance or Repa	air		
Least impacting satellite out:	99.98%	99.17%	
Average satellite out:	99.93%	97.79%	
Most impacting satellite out:	99.83%	97.63%	
Two Satellites Out for Maintenance or Rep	pair		
Least impacting 2 satellites out:	99.93%	98.21%	
Average 2 satellites out:	99.64%	95.71%	
Most impacting 2 satellites out:	98.85%	91.08%	
Three Satellites Out for Maintenance or Repair			
Least impacting 3 satellites out:	99.89%	97.13%	
Average 3 satellites out:	99.03%	93.38%	
Most impacting 3 satellites out:	95.87%	83.92%	

### 3.2 Expected Service Availability Characteristics

Table 3-1 defines what service availability characteristics will be like for a given satellite outage condition. Service availability projections over time may be generated by applying the information in Table 3-1 to expected satellite control operations scenarios. A satellite control operations scenario is based upon a conservative estimate of satellite maintenance activity frequency and duration. Satellite maintenance actions requiring service downtime include periodic cesium frequency

standard maintenance, station keeping maneuvers to maintain orbits within toerances, and responses to component failures. Given current routine maintenance requirements and component failure expectations, on average four or fewer satellites should be removed from service over any 30 day period. Once a satellite is removed from service, it is assumed that it will be down for no more than 24 hours.

The first service availability scenario to be defined represents a worst-case 30 day period. A summary of this scenario is provided in Table 3-2. The scenario is considered to be worst-case from two perspectives: it includes a day with three satellites removed from service, and it includes a total of four satellite-down days. The three satellite-down scenario is based upon the simulaneous removal of two satellites for routine maintenance, accompanied with a component failure on a third satellite. Worst case global service availability on a day with three satellites removed from service is 95.87%; the associated worst case regional service availability is 83.92%. The resulting 30-day service availability values range from 99.85% to 99.99%, depending on which satellites make up the four which experience downtime. The service availability service standard was established based upon this scenario, to ensure that the system can support standard conpliance.

Table 3-2. Example of 30-Day Global Service Availability with Component Failure on Worst Day

Ops Scenario Condition	Best Case	Average Case	Worst Case
1 Day - 3 satellites down	0.9989	0.9903	0.9587
1 Day - 1 satellite down	0.9998	0.9993	0.9983
28 Days - No satellites down	1.0	1.0	1.0
Average Daily Availability	99.99%	99.97%	99.85%

The second service availability scenario is shown in Table 3-3, and represents what may be considered to be a more common 30 day interval. In this scenario, three satellites were renoved from service for up to 24 hours, each on separate days. Typical satellite maintenance operations are conducted on one satellite at a time, which means that the removal of two satellites for maintenance at the same time will be a rare occurrence. Global service availability on a day where the worst case satellite is removed from service is 99.85%; the associated worst case regional service availability is 97.63%. The resulting 30-day service availability values do not change much between the best and worst cases, with the worst case value being 99.98%.

Table 3-3. Example of 30-Day Global Service Availability without Component Failure

Ops Scenario Condition	Best Case Average Case		Worst Case
3 Days - 1 satellite down	0.9998	0.9993	0.9985
27 Days - No satellites down	1.0	1.0	1.0
Average Daily Availability	99.99%	99.99%	99.98%

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## **SECTION 4.0 Service Reliability Characteristics**

This section defines conservative expectations for GPS service reliability performance. These expectations are based upon observed accuracy characteristics, the GPS service failure history to date, long-term failure rate projections, and current system failure response capabilities. The user is provided with information which indicates expected failure rates and their effects on a global and regional basis.

#### 4.1 Reliability Threshold Selection

As defined in Section 1.3, service reliability is the measure of how consistently GPS horizontal error levels can be maintained below a specified reliability threshold. The selection of an appopriate value for this threshold is based upon an assessment of normal accuracy characteristics. A description of normal accuracy characteristics is provided in Section 5.2, which contains expected error statistic variations and distributions. The value must be larger than the practical limit on normal GPS horizontal performance. The largest horizontal error that can be experienced under normal operating conditions, with a PDOP constraint of 6, is approximately 400 meters. A value of 500 meters was chosen as the reliability threshold because it is sufficiently outside the normal GPS SPS accuracy envelope to avoid a false alarm condition, and because it should serve as a usable input to aviation plans for phases of flight down to terminal area operations.

Given a horizontal error reliability threshold, a corresponding Not-To-Exceed (NTE) ranging error threshold may be defined that bounds the SPS horizontal error within the specified threshold for a specified range of position solution geometries. A ranging error threshold is used in the service failure detection process as opposed to a position error threshold, due to the practical difficulties associated with monitoring position solutions on a global basis. A ranging error threshold of 150 meters will provide a 500 meter bound on the maximum predictable horizontal error, given a maximum Horizontal Dilution of Precision (HDOP) of 4.

#### 4.2 GPS Service Failure Characteristics

A service failure is defined to be a condition where the positioning service is exhibiting time odered error behavior which is atypical. An occurrence of this behavior is directly due to a failure somewhere in the GPS ranging signal control and generation process.

Service failures are classified into two categories: minor and major. A minor service failure is defined to be a departure from the normal ranging signal characteristics in one of the following ways:

- A statistical departure from nominal system ranging accuracy which does not cause the instantaneous SPS ranging error to exceed 150 meters.
- A navigation message structure or content violation which does not impact the minimum SPS receiver's navigation message processing capabilities.

A major service failure is defined to be a departure from the normal ranging signal characteistics in a manner which can cause a reliability or availability service failure. A major service failure is

defined to be a departure from the normal ranging signal characteristics in one of the following ways:

- A statistical departure from nominal system ranging accuracy which causes the SPS instantaneous ranging error to exceed 150 meters, or
- An SPS ranging signal RF characteristic, navigation message structure or navigation message contents violation that impacts the SPS receiver's minimum ranging signal eception or processing capabilities.

The characteristics of a service failure and the factors which affect service reliability are listed lelow. Each is discussed in more detail in the following sections.

- Ranging signal failure frequency.
- Failure duration.
- Failure magnitude and behavior.
- Distribution of user population around the globe.
- Probability that the failed satellite is used in the position solution.
- Effect that the failure has on the position solution, given the failed satellite's contribution to solution geometry and the receiver's response to the failure condition.

#### 4.2.1 Failure Frequency Estimate

The GPS satellite positioning service failure history over the past several years indicates a very low service failure rate (excluding Block I satellites). However, when a service failure does occur, it can result in extremely large position and/or velocity errors. This behavior will typically persist until action is taken to remedy the problem.

Based upon an historical assessment of Block II satellite and Control Segment failure characteritics, GPS should experience no more than an average of three major service failures per year (excluding Block I satellites). This failure rate estimate is conservative -- expectations are on the order of one per year, based upon projected navigation payload componentreliabilities and the assumption that action will be taken to switch redundancy configurations if early indications of an imminent failure are detected. An allocation of three per year allows for a possible increase in service failures as the Block II satellites reach the end of their operational life expectancy.

#### 4.2.2 Failure Duration Estimate

The duration of a failure is a function of the following factors:

- · Control Segment monitor station coverage,
- Control Segment monitor station, communications and Master Control Station availability,
- Master Control Station failure detection efficiency and timeline,
- Timeline for correcting the problem or terminating the failed satellite's service, and
- Control Segment ground antenna coverage and availability.

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The combination of these factors results in a conservative system operator response timeline on the order of no more than six hours. In most cases the response to a failure will be much more prompt, but with any complex system such as the Control Segment, allowances must be made for varying system resource status and operational conditions. The nominal failure response time, taking into consideration a favorable combination of the above factors, is on the order of 10-30 minutes.

#### 4.2.3 Failure Magnitude and Behavior

GPS is designed to be fault tolerant -- most potential failures are either caught before they mainfest themselves, or their effects are compensated for by the system. The only failures to which the system seems susceptible are of two types:

- Insidious, long-term (day or more to manifest themselves) performance deviations, or
- Major (catastrophic), almost instantaneous failures.

Insidious failures do not propagate very quickly -- failures of this type experienced to date have not affected the GPS ability to support SPS accuracy performance standards. Insidious failures are typically due to a problem in the ephemeris state estimation process.

Major failures are due almost exclusively to satellite code and carrier generation hardware faiures. These failures in general result in very rapid ranging error growth -- range errors can grow to several thousand meters in a very short period of time. One example of a failure of this type will begin with a phase jump of indeterminate magnitude, followed by a large ramp or increased noise consistent with the behavior of a quartz oscillator.

#### 4.2.4 User Global Distribution and Failure Visibility

For the purposes of reliability performance standard definition, the effect of a service failure is not weighted based upon user distribution -- a uniform distribution of users over the globe is sumed.

Given a maximum failure duration of six hours, approximately 63% of the Earth's surface will have a failed satellite in view for some portion of the failure. The average amount of time that the failed satellite will be in view for those locations which can see it is approximately three hours.

#### 4.2.5 Satellite Use in the Position Solution

Given a 24 satellite constellation, an average of eight satellites will be in view of any user on or near the Earth. The satellite visibility distribution for the nominal 24 satellite constellation is shown in Figure 2-1. With all satellites weighted equally, the probability of a failed satellite being in the position solution of any user located within the failure visibility region is 50%. Equal weighting is considered to be a reasonable assumption for use in global reliability computations. However, in the worst-case individual site computation it must be assumed that the receiver is tracking and using the failed satellite for the duration of the satellite visibility window.

#### 4.2.6 Failure Effect on Position Solution

Given the nature of catastrophic failures, it must be assumed that the inclusion of a satellite in the position solution will induce a service reliability failure independent of the satellite's geometric contribution. Some receivers will be capable of detecting and rejecting large instantaneous changes in a range residual which are indicative of a major service failure. The minimum SPS

receiver represented in the Signal Specification is not however required to have this capability. For the purposes of service reliability standard definition, it must be assumed that if the receiver is capable of tracking the failed satellite and it supports the nominal position solution geometry, the receiver will use it in the position solution.

#### 4.3 Expected Service Reliability Characteristics

When the system is performing nominally and the receiver design meets the minimum usage conditions established in Section 2.2 of the Signal Specification, predictable horizontal error will never reach the service reliability threshold. Service reliability on those days where GPS does not experience a major service failure will be 100%.

The estimated maximum of three major service failures per year, coupled with a maximum duation of six hours each, yields a maximum of 18 service failure hours per year. The worst-case site on the globe will be the place where all 18 service failure hours are observed and the failed sate lites are used in the position solution. For this worst-case condition, the daily average service relability over a one year period will be no worse than 99.79%. The equivalent global daily average will be no worse than 99.97%.

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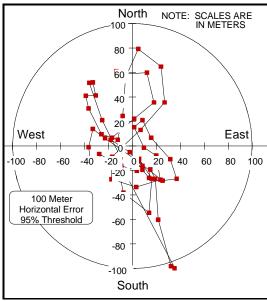
## **SECTION 5.0 Accuracy Characteristics**

This section describes GPS position solution time ordered behavior, and defines expected error statistic characteristics for four different aspects of accuracy: predictable, repeatable, relative and time transfer. The user is provided with information describing GPS accuracy daily variations and accuracy as a function of user location.

One of the underlying assumptions implicit in the definition of the accuracy performance ewelope is that satellite ranging error statistics across the constellation are approximately ergodic. In reality, this may not be the case for several reasons. Regardless of the variations in ranging performance across the constellation, positioning and timing performance will be no worse than it is represented by the accuracy performance standards.

#### 5.1 Positioning Error Time Ordered Behavior

Unlike a system such as LORAN-C, GPS position solution errors change considerably over time at any given location. Figure 5-1 demonstrates typical position solution horizontal coordinate changes from minute-to-minute over a one-hour interval, as they would be seen by a user located at the coordinate crosshairs. Based upon observed system behavior, the horizontal postion estimate shifts about one meter every second on aveage. A statistical behavior pattern begins to emerge when the observation window is widened to 24 hours (the sample interval specified in the accuracy performance standard). As shown in Figure 5-2, horizontal errors are centrally grouped, with a few outliers near and beyond the 95% performance standard circle. Excursions beyond the 100-meter circle are infrequent, and they seldom last more than a minute.



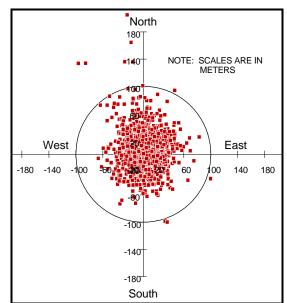
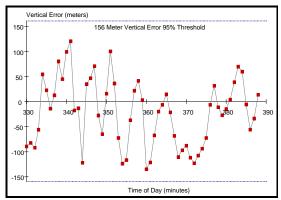


Figure 5-1. Horizontal Errors over 1 Hour

Figure 5-2. Horizontal Errors over 24 Hours

Changes in vertical coordinate estimates are generally larger than those in the horizontal plane, due to the nature of the position solution geometry. Figure 5-3 provides an example of how vertical errors change from minute-to-minute over a one-hour interval. Based on observed system behavior, the vertical position estimate shifts about 1.5 meters every second on average.

The 24-hour plot of vertical errors in Figure 5-4 show a central grouping, with few deviations beyond the 156 meter line.



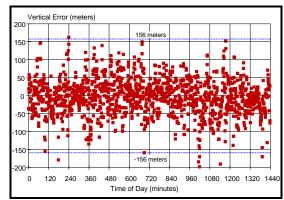


Figure 5-3. Vertical Errors over 1 Hour

Figure 5-4. Vertical Errors over 24 Hours

Instantaneous position estimate changes, on the order of tens of meters, may be observed at times during a transition between satellites used in the position solution. This transient behavior is due to an abrupt change in solution geometry, combined with differences in ranging errors between the old and new satellite(s). On those occasions where several large jumps in the position solution are observed over the course of a few minutes, this behavior is generally due to multiple changes in the receiver's satellite selection.

#### 5.2 Predictable Accuracy Characteristics

As mentioned in Section 1.3.4 of this Annex, predictable accuracy statistics vary as a function of sample interval and user location. The following discussions focus on both of these factors, and their implications on the ability of GPS to support SPS predictable accuracy requirements. The discussion concludes with a description of expected GPS SPS predictable accuracy distribution characteristics.

#### 5.2.1 Daily Variations in Positioning Errors

GPS accuracy requirements are stated in terms of 24-hour measurement intervals. Even in steady-state operations however, the full range of GPS position error behavior can not be experienced over 24 hours at any given site. As a result, error statistics over any set of 24-hour intervals will vary. Measured average daily variations of GPS 24-hour 95% error statistics over 30 days of steady-state operations are as follows:

• East: 15%

• North: 14%

Vertical: 10%

Horizontal: 10%

In the event that ranging error statistical behavior changes for one or more satellites over a given interval, larger variations than those listed above may be observed by the user.

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#### 5.2.2 Geographic Variations in Positioning Errors

GPS predictable accuracy performance is specified in terms of global horizontal and vertical errors over 24 hours for any location on or near the Earth. Performance will however vary considerably as a function of user location. The purpose of the following discussion is to characterize how GPS predictable accuracy varies as a function of user latitude and longitude. Stated values will hold true for steady-state constellation operations, with all satellites providing similar range error characteristics. Error estimates are based upon 24-hour measurement intervals, with at least 30 days of steady-state operations. Figure 5-5 provides a summary of GPS performance variation as a function of user latitude.

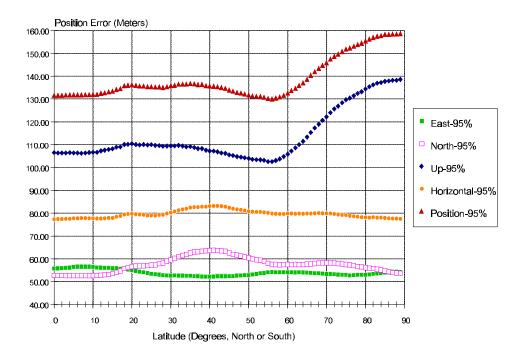


Figure 5-5. GPS Accuracy as a Function of Latitude

Due to the nature of the satellite constellation design, GPS exhibits hemispherical symmetries in coverage and accuracy attributes. GPS error behavior is approximately symmetric between the Northern and Southern hemispheres. GPS latitude-dependent errors for any given longitude in the Southern Hemisphere are phased 90 degrees with respect to Northern Hemisphere longitudinal error characteristics. GPS longitude-dependent errors in any half of a hemisphere (North or South) are approximately symmetric with the other half of the same hemisphere.

Average longitudinal variations in east, north, and vertical 95% errors are less than 5% for any given latitude (excluding transient areas of degraded coverage). Average longitudinal variations in horizontal and (three-dimensional, or 3D) position 95% errors for any given latitude are less than 2% (excluding transient areas of degraded coverage). Degraded coverage areas will cause spikes in the latitude-dependent error curve. The magnitude of a spike depends on the service availability characteristics for the time interval of interest. Given that all 24 satellites are available, the largest expected growth in north, east or horizontal 24 hour 95% error values for a site affected by degraded coverage is 12% above the nominal value for that latitude (given a PDOP constraint of 6). Vertical and 3D position 95% error values can increase by as much as 31% for those areas affected by a coverage degradation (once again, given a PDOP constraint of 6).

Given the coverage and service availability characteristics discussed in Sections 2 and 3, these degradation areas are not expected to occur very often.

North errors vary considerably more as a function of latitude than do east errors. 95% east errors are generally larger than 95% north errors between ± 18° latitude. After 18°, the north error statistic can become as much as 22% larger than the east error statistic. Vertical error grows 3% between 0° and ± 21° latitude, gradually decreases 7% between 20° and 56°, and then grows 35% between 56° and 90°. This growth behavior after 56° is due to the fact that the maximum satellite elevation angle decreases steadily as latitude increases beyond the nominal satellite orbit inclination of 55°.

Horizontal error grows 7.5% between 0° and  $\pm$  43° degrees latitude, and then gradually decreases 7.2% between 43° and 90°. 3D position error statistics follow the same general trend as vertical error statistics, due to vertical error dominance and almost constant horizontal error behavior as a function of latitude.

#### 5.2.3 Expected Error Distribution Characteristics

Error distributions provide a convenient means of summarizing predictable accuracy characteristics. However, the definition of any GPS position error distribution must be caveated with the fact that performance varies as a function of sample interval and location, as was discussed in the preceding sections. The error distributions provided in this section are based upon measured data from the GPS Control Segment monitor These monitor stations are in stations. general located close to the equator, so the local axis error distributions will deviate from their representation in Figure 5-6 as user latitude increases. The horizontal error distribution shown in Figure 5-7 is fairly representative of expected performance, irregardless of latitude. The distributions in Figures 5-6 and 5-7 are all generated using three months' worth of data, so a user can expect to see daily variations with respect to these distributions in accordance with the discussion in Section 5.2.1.

The empirical error distributions are overlaid with Gaussian distributions, as a basis for comparison with theoretical expectations. The theoretical distributions were generated using the means and standard deviations of the empirical datasets. These figures that GPS position indicate error characteristics are not necessarily Gaussian.

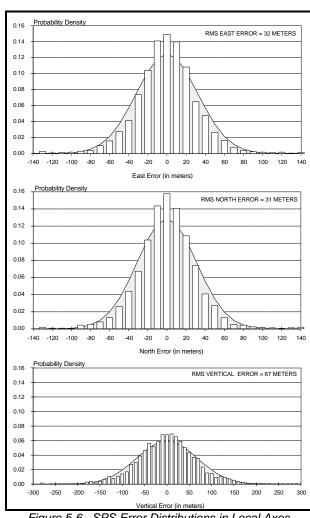


Figure 5-6. SPS Error Distributions in Local Axes

Annex A to the SPS Signal Specification establishes additional predictable accuracy standards of 300 meters horizontal error and 500 meters vertical error, both with a 99.99% confidence over any 24 hour interval, at any given

Page B-18 2nd Edition location in the world. Based upon observed distribution characteristics, these standards will be met as long as the system does not experience a service failure.

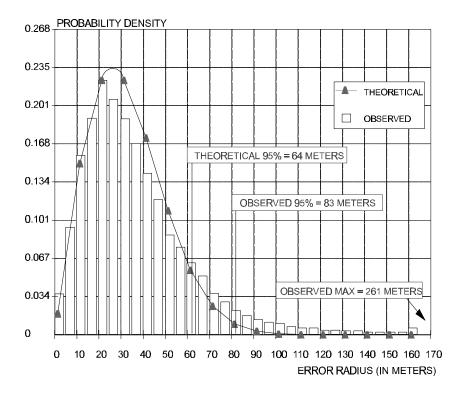


Figure 5-7. The Nominal SPS Horizontal Error Distribution

### 5.3 Repeatable Accuracy Characteristics

Repeatable accuracy statistics vary primarily as a function of the time between position measurements. The error in general grows as the time between measurements increases, until the interval reaches approximately 4 minutes. After 4 minutes, the repeatable error statistics are essentially independent of time between measurements. This behavior is reflected in Figure 5-7, where the RMS horizontal repeatable error grows to approximately 53 meters before its behavior stabilizes. Horizontal 95% repeatable accuracy is on the order of 105 meters, and vertical 95% repeatable accuracy is on the order of 165 meters at the equator. Based upon this performance, repeatable accuracy performance will remain within the performance standards, irrespective of daily or geographic variations.

## **5.4 Relative Accuracy Characteristics**

The graph in Figure 5-9 below shows how errors in the measurement of the relative vector be tween two receivers tracking the same satellites grow, as a function of time between the two receivers' measurements. The receivers must be within 40 kilometers of each other, in order to

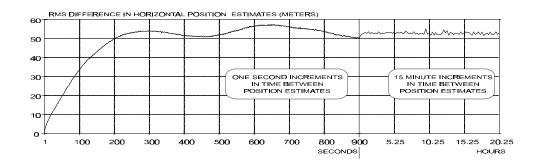


Figure 5-8. Repeatable Accuracy as a Function of Time between Position Estimates

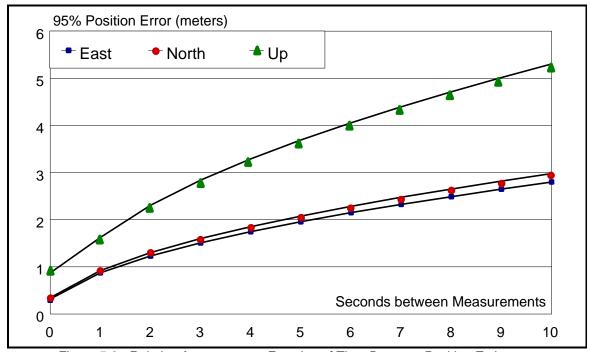


Figure 5-9. Relative Accuracy as a Function of Time Between Position Estimates

experience performance consistent with the performance standard. The 40 kilometer value is considered to be a practical limit on the distance between receivers for relative positioning implementations, based upon a strict conformance with the relative positioning definition and performance standard values established in the Signal Specification. The limit is based upon the fact that satellite tracking coordination to support optimum position solution geometries (PDOP  $\leq$  6) becomes increasingly difficult for receivers further than 40 kilometers apart.

Users need to be aware of the fact that the accuracy performance standards are based upon signal-in-space error characteristics, and do not take into consideration receiver contributions to error statistics. The other aspects of accuracy are not affected by this distinction in a practical sense, since the receiver error contribution is very small relative to the signal-in-space error. In the case of relative accuracy however, receiver noise characteristics become the dominant error source. User relative accuracy performance will depend significantly on consistency in the two

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receivers' designs, and coordination of position solution satellite selection and generation timing. Users may experience performance which is consistent with Figure 5-9, or 95% horizontal relative accuracies as large as 6 meters and 95% vertical relative accuracies as large as 9 meters depending on the receivers used.

#### 5.5 Time Transfer Accuracy Characteristics

Time transfer accuracy based upon the output of the position solution is a function of SPS timing errors with respect to GPS time, and GPS time scale errors with respect to Universal Coordinated Time (UTC) as it is maintained by the United States Naval Observatory. Current satellite SPS timing errors with respect to GPS time are on the order of 75 nanoseconds RMS (including propagation effects). The GPS Control Segment consistently manages GPS time coordination with UTC to better than 30 nanoseconds.

When the combined GPS time prediction error and GPS-UTC time synchronization errors are mapped into the position solution, the RMS time transfer error is on the order of 110ns. The 95% UTC time transfer error should not exceed 250 nanoseconds, based upon measurements of the position solution time offset. The 340 nanosecond performance standard defined in Annex A provides the Control Segment with flexibility in the management of the GPS time scale.

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# GLOBAL POSITIONING SYSTEM STANDARD POSITIONING SERVICE SIGNAL SPECIFICATION

#### ANNEX C

## MEANS OF MEASURING GPS PERFORMANCE



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#### **SECTION 1.0 Introduction**

Performance standards can not be considered to be valid unless they are quantifiable, and the ability of GPS to meet them can be consistently measured. The first criteria is met with the specific definitions contained in Annex A, of GPS performance standards and the conditions under which they will be met. The ability to meet the second criteria, to consistently measure GPS performance, depends upon the establishment of methodologies which can be universally applied to the performance evaluation process.

#### 1.1 Purpose

The purpose of this Annex is to define measurement methodologies for evaluating GPS per formance against the established performance standards. The measurement methodologies de fined in this Annex consist of three parts: measurement groundrules, minimum equipment re quirements, and measurement algorithms.

#### 1.2 Scope

The measurement methodologies are designed specifically to assess performance in a manner which is consistent with the performance standards defined in Annex A. The methodologies do not address any aspects of performance beyond those established by the performance standards.

Performance measurement processes defined in this Annex are restricted to individual site performance assessments. The single point performance standards are the success criteria for each assessment. The global performance standards represent conservative average performance values for any arbitrary point on or near the surface of the Earth. In this capacity, the civil user can apply the global performance standards to provide an indication of GPS performance relative to an "average" location.

The methodologies defined in this Annex support general positioning and timing performance measurements against the performance standards. The algorithms provided in this Annex may not be suitable for use with more specific applications of the SPS, such as surveying or differential GPS operations.

Note that all measurement algorithms were developed based upon the spherical Earth assumption. A tradeoff was made in the algorithm designs, between algorithm complexity and a degradation in precision. The spherical Earth algorithms are simple to implement, and provide a minimal degradation in generic position error measurement precision, particularly when compared to the magnitude of the measurements being processed. Application of these algorithms will result in position error measurements which vary less than 0.4% with respect to results obtained using an oblate Earth model. Users who are concerned with measuring GPS performance for applications beyond the scope of this Signal Specification may find that they require the additional precision which more sophisticated algorithms will provide.

The user is cautioned not to apply these algorithms within approximately 100 kilometers of the North or South poles, without making provisions to support the special case of polar measurements.

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GPS measurement methodologies are defined using the metric system. Angular quantities are expressed in degrees.

Error distribution generation is not explicitly addressed, nor are distribution characteristics specified in the measurement methodologies. However, any desired empirical accuracy distribution is easily generated from the raw measurement data by sequentially sorting and binning the data in appropriately sized bins.

#### 1.3 References

The following references were used in the development of the GPS performance measurement methodologies:

- Department of Defense World Geodetic System 1984, Its Definition and Relatio nships with Local Geodetic Systems, DMA Publication TR-8350.2 (unlimited distribution), Second Edition, September 1, 1991.
- Clyde R. Greenwalt and Melvin E. Shultz, *Principles of Error Theory and Cartographic Applications*, United States Air Force Aeronautical Chart and Information Center Publication ACIC Technical Report No. 96 (unlimited distribution), February 1962.
- Gerald J. Hahn and William Q. Meeker, *Statistical Intervals: A Guide For Practitioners* (New York: John Wiley & Sons, Inc., a Wiley-Interscience Publication, 1991).

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## **SECTION 2.0 Performance Measurement Groundrules**

This section defines groundrules for measuring and evaluating any aspect of GPS performance against the performance standards. Failure to follow the groundrules may lead to erroneous performance measurement results.

**GROUNDRULE 1:** All performance measures are defined with respect to the WGS-84 ellipsoid and associated ECEF coordinate systems. Errors with respect to the geoid, other ellipsoids and their associated terrestrial coordinate systems, or with respect to terrain features are not defined.

**GROUNDRULE 2:** Methodologies do not take into consideration the effects of local obscura above specified mask angles.

**GROUNDRULE 3:** Methodologies are designed to measure GPS space and control segment contributions to SPS performance. Methodologies therefore consider nominal assumed GPS receiver characteristics as they are defined in the Signal Specification central document. The effects of aiding or augmentations to the basic GPS signal are not considered.

**GROUNDRULE 4:** To simplify algorithm usage, all measurement methodologies use a one-second interval between samples. Maximum intervals between measurements for each performance parameter are provided below, for those who wish to reduce sample rates to minimize data processing and storage requirements. If the sample rate is reduced for an algorithm which provides an input to another algorithm, use the previous measurement value until the next sample. For example, if the service availability interval between samples is increased to 30 seconds, use the previous service availability measurement to support service reliability measurements until the next service availability measurement.

Performance ParameterMaximum Interval between SamplesCoverage30 SecondsService Availability30 SecondsService ReliabilityFour SecondsAccuracy - 95% Confidence Value60 SecondsAccuracy - 99.99% Confidence ValueFour SecondsAccuracy - Not-to-Exceed ValueOne Second

Table 2-1. Maximum Intervals between Samples

**GROUNDRULE 5:** To ensure consistency in comparisons of results between measurements taken by independent groups, sample collection start time in methodologies is defined to be 0000Z. Since performance requirements are not stated in terms of specific measurement interval start and stop times, start times other than 0000Z may be used at the convenience of users who are not concerned with performance comparisons.

**GROUNDRULE 6:** In general, assume that at least 90% of the available data points must be collected over the sample interval to provide a representative performance assessment for a given parameter.

**GROUNDRULE 7:** To ensure that GPS civil performance measurement datasets are consistent with performance standard definitions, all measurements must be taken using a system which meets the minimum requirements for a *GPS Measurement System*. Measurements taken using a system which does not meet the minimum requirements may not support representative system performance evaluations.

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# SECTION 3.0 Measurement System Minimum Requirements

Standardized measurement methodologies require a minimum set of measurement capabilities. A system which provides these minimum capabilities is referred to as a *Measurement System*. This section defines the minimum requirements for establishing a Measurement System.

## 3.1 Measurement System Configurations

Three different types of generic Measurement Systems are defined, to support varying performance measurement needs. Each type requires a successively more sophisticated configuration.

- Measurement System Type 1: Coverage and Availability Measurement
- Measurement System Type 2: Position Accuracy and Service Reliability Measurement
- Measurement System Type 3: Time Transfer and Range Domain Measurement

Figure 3-1 below illustrates an example configuration of a Type 3 Measurement System.

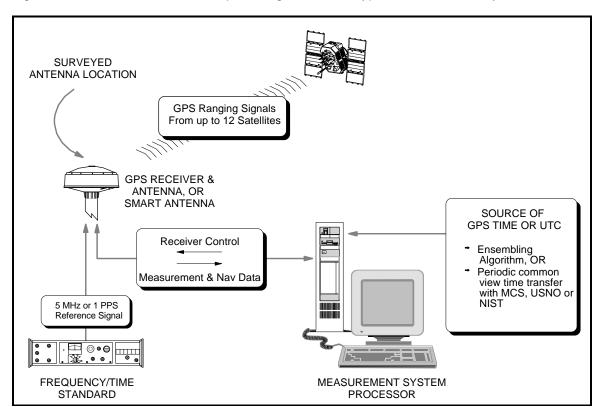


Figure 3-1. Example of Type 3 Measurement System Configuration

Measurement System Type 1 requires that an SPS positioning receiver be tied to a Measurement System processor. The processor controls the receiver configuration, and receives and processes measurement and navigation message data coming from the receiver.

Measurement System Type 2 has the same configuration as Type 1, with the addition of placing the receiver antenna on a surveyed benchmark.

Measurement System Type 3 has the same configuration as Type 2, with the following additions:

- A frequency/time standard to drive receiver range measurement time tags, and
- A mechanism, algorithm or process to synchronize Measurement System time with GPS time and/or UTC.

## 3.1.1 The Measurement System Processor

The Measurement System processor must support the following functions:

- Receiver control message generation/transmission.
- Measurement and navigation message data reception.
- Generation of raw data files containing performance standard measurement data.

Performance measurement processing can be conducted on the processor, or raw data may be exported for computations on an external processor.

The Measurement System processor supports coverage and availability measurements in the following fashion:

- Generate satellite visibility files as a function of time and mask angle based upon the transmitted almanac.
- Compute the optimum solution geometry based upon the minimum PDOP.
- Read the ephemeris for each of the selected satellites. Check satellite health and for the presence of any flags indicating that the satellite should not be used; if a satellite is unhealthy, update availability files to reflect satellite status, and recompute the optimum solution geometry.

The Measurement System processor supports predictable accuracy and reliability measurements by computing position errors with respect to the surveyed benchmark. Re peatable accuracy measurements are supported through the differencing of position or predict able error vectors. Relative error measurements are supported through the use of two Meas urement Systems, or two receivers tied to the same Measurement System. Time transfer error measurements are supported by using a time source tied to UTC as the timing input to the pe sition solution process. Range domain measurements are supported by using a time source tied to GPS time as the timing input to the receiver pseudo range measurement process. This tie effectively allows the user to isolate the user's time bias during the range residual computation.

#### 3.1.2 The Measurement System Frequency/Time Standard

The Type 3 Measurement System requires a stable frequency (5 or 10 MHz, for example) and/or time pulse (1 PPS) output to the receiver or a time interval counter. The frequency/time source must exhibit the following short-term stability requirements, at a minimum. An uninterruptable power supply is recommended for long-term observations.

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Table 6 1: Mededrement Cyclem enert Term Clabinty Requirements	
Averaging Interval	Time Domain Stability $-\sigma_{v}(\tau)$
1 second averaging:	1 x 10 <sup>-9</sup> s/s
10 second averaging:	1 x 10 <sup>-10</sup> s/s
100 second averaging:	1 x 10 <sup>-11</sup> s/s
1,000 second averaging:	1 x 10 <sup>-12</sup> s/s
10,000 second averaging:	1 x 10 <sup>-12</sup> s/s

Table 3-1. Measurement System Short-Term Stability Requirements

### 3.1.3 The Measurement System GPS Receiver

The GPS receiver selected for use in a Measurement System must provide the following capa bilities to support coverage and availability performance assessments:

- A communications interface compatible with the selected Measurement System processor and time/frequency standard.
- Output the PRN numbers of the satellites being tracked up to once per second.

All satellite-in-view tracking is desirable but not mandatory. The receiver antenna must be installed at a surveyed location for Type 2 or 3 Measurement Systems. The service standard assumes a survey accuracy of at least 1 meter (1 $\sigma$ ) in each local coordinate axis. The GPS receiver must provide the following minimum capabilities to support position accuracy and service reliability performance assessments:

- Measurement data outputs up to once per second.
- Navigation message data output upon request or upon detection of an update.

The GPS receiver, working in concert with the Measurement System processor and frequency/time standard, must provide the following minimum capabilities to support time transfer accuracy performance assessments:

- Measurement timing must be based upon satellite ensemble time, an external reference or a selected satellite reception time tag -- regardless of the method used, measurement time tag precision with respect to reference time must be no worse than 10 ns RMS.
- Range or range residual measurement outputs up to once per second.

# 3.2 Minimum Position Error Measurement Processing Requirements

This section defines the specific process for computing instantaneous position solution error vectors. Many GPS receivers use sophisticated processing techniques such as range residual smoothing, velocity aiding, Kalman filters, all-in-view satellite solutions, etc. The minimum performance statistics are however based upon mapping instantaneous range residuals into a user position residual vector through the linearized position solution from a stationary, surveyed location. This process will result in the measurement of positioning and timing error characteristics which a receiver designed in accordance with the minimum requirements established in the Signal Specification can reasonably be expected to experience.

**STEP 1.** Select optimum four satellites based upon minimum PDOP. Update every five minutes, or whenever a satellite being used in the solution sets. See Step 6 for computation of the **K**-matrix terms. Note that the K terms used in the DOP computation may be computed using either the almanac or each satellite's ephemeris.

PDOP = 
$$\sum_{i=1}^{3} \sum_{j=1}^{4} K_{ij}^{2} \Big|_{2}^{1}$$

**STEP 2.** Measure the pseudo range to each satellite. Each of the four measurements must have a reception timetag within  $\pm 0.5$  seconds of the solution time. The reception timetag is based upon Measurement System time, and the transmission timetag is based upon satellite time.

$$PR_{\text{measured}}^{\text{svi}}(t_{\text{received}}) = c(t_{\text{received}}^{\text{svi}} - t_{\text{transmitted}}^{\text{svi}})$$

Note that the pseudo range must be corrected based upon propagation path and timing error effect corrections defined in the SPS Signal Specification. c equals the speed-of-light in a vacuum (299,792,458 meters/second) as defined in the SPS Signal Specification, Section 2.5.1.

STEP 3. Read the ephemeris for each of the four satellites, in accordance with the SPS Signal Specification. Compute each satellite's ECEF position at the time of transmission. Apply the Earth rotation correction terms defined in the SPS Signal Specification to the site coordinates. Compute the predicted pseudo range for each satellite. (Readers should note that this predicted pseudo range computation yields what is alternatively identified as the *geometric range* in the SPS Signal Specification, Section 2.5.4.2).

$$\mathsf{PR}^{\mathsf{svi}}_{\mathsf{predicted}} = \left\| \overline{\mathsf{R}}^{\,\mathsf{svi}}_{\mathsf{predicted}} (t_{\,\mathsf{transmitted}}) - \overline{\mathsf{R}}_{\mathsf{site}} \right\|$$

where: 
$$\overline{R}_{predicted}^{svi}(t_{transmitted})$$
 = Estimated position of ith satellite at time of transmission  $\overline{R}_{site}$  = Location of receiver antenna, corrected for Earth rotation effects

STEP 4. Compute the range residual for each satellite. Given that \$\mathbb{t}\_{\text{eceived}}\$ is within \$\pm 0.5\$ seconds of the position solution time \$\mathbb{t}\_{\kappa}\$, the range residual is associated with the \$\mathbb{k}^{\text{th}}\$ position solution time. NOTE: readers should not confuse the variable \$\mathbb{t}\$ as it is used in this Annex with its usage in the Signal Specification (Section 2.5.4, Table 2-15). \$\mathbb{t}\$ is used in the Signal Specification to define the difference between epoch time and current GPS time; it is used in this Annex to define an arbitrary \$\mathbb{k}^{\text{th}}\$ position solution time.

$$\Delta r_{\text{svi}}\!\left(t_{\text{k}}\right)\!=\!\text{PR}_{\text{measured}}^{\text{svi}}\!\left(t_{\text{received}}\right)\!-\!\text{PR}_{\text{predicted}}^{\text{svi}}\!\left(t_{\text{received}}\right)$$

STEP 5. Compute the position solution geometry matrix G, and rotate it into local coordinates. The G-matrix (defined below) is composed of four row vectors: one for each of the four satellites in the position solution. Each row vector contains the x, y, z and time coordinate direction cosines associated with one of the four satellite-to-user vector geometries, as they are defined in the WGS-84 ECEF coordinate system.

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$$\mathbf{G}_{XYZ} = \begin{bmatrix} \frac{X_{site} - X_{sv1}}{R_{sv1} - ct_b} & \frac{Y_{site} - Y_{sv1}}{R_{sv1} - ct_b} & \frac{Z_{site} - Z_{sv1}}{R_{sv1} - ct_b} & 1 \\ \frac{X_{site} - X_{sv2}}{R_{sv2} - ct_b} & \frac{Y_{site} - Y_{sv2}}{R_{sv2} - ct_b} & \frac{Z_{site} - Z_{sv2}}{R_{sv2} - ct_b} & 1 \\ \frac{X_{site} - X_{sv3}}{R_{sv3} - ct_b} & \frac{Y_{site} - Y_{sv3}}{R_{sv3} - ct_b} & \frac{Z_{site} - Z_{sv3}}{R_{sv3} - ct_b} & 1 \\ \frac{X_{site} - X_{sv3}}{R_{sv3} - ct_b} & \frac{Y_{site} - Y_{sv3}}{R_{sv3} - ct_b} & \frac{Z_{site} - Z_{sv3}}{R_{sv3} - ct_b} & 1 \\ \frac{X_{site} - X_{sv4}}{R_{sv4} - ct_b} & \frac{Y_{site} - Y_{sv4}}{R_{sv4} - ct_b} & \frac{Z_{site} - Z_{sv4}}{R_{sv4} - ct_b} & 1 \end{bmatrix} = \begin{bmatrix} G_{x}^{sv1} & G_{y}^{sv1} & G_{z}^{sv2} & 1 \\ G_{x}^{sv2} & G_{y}^{sv2} & G_{z}^{sv2} & 1 \\ G_{x}^{sv3} & G_{y}^{sv3} & G_{z}^{sv3} & 1 \\ G_{x}^{sv4} & G_{y}^{sv4} & G_{z}^{sv4} & 1 \end{bmatrix}$$

where:  $\{x_{site}, y_{site}, z_{site}\} = Station location in Cartesian coordinates$ 

 $\{x_{svi}, y_{svi}, z_{svi}\} = i\frac{th}{s}$  satellite position coordinates at transmission time based upon navigation message contents

R<sub>svi</sub> = Estimated range from user to ith satellite -- can use predicted pseudo range.

ct<sub>b</sub> = Bias between Measurement System time and GPS time, multiplied by the speed of light -- time should be managed such that the bias value is nominally zero.

Use the coordinate rotation matrix S to rotate each of the G-matrix row vectors into local coordinates. The S-matrix is defined below.

$$\mathbf{S} = \begin{bmatrix} -\sin\lambda_{site} & \cos\lambda_{site} & 0 & 0 \\ -\sin\phi_{site}\cos\lambda_{site} & -\sin\phi_{site}\sin\lambda_{site} & \cos\phi_{site} & 0 \\ \cos\phi_{site}\cos\lambda_{site} & \cos\phi_{site}\sin\lambda_{site} & \sin\phi_{site} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where:  $\{\phi_{\text{site}}, \, \lambda_{\text{site}}\}\ =\ \text{Station latitude}$  and longitude in local coordinates

The G-matrix row vector rotation is defined below. The result of the rotation is a newG-matrix, defined with respect to local coordinate axes. Note that the time axis remains invariant through the rotation process.

$$\mathbf{G}_{\text{enu}} = \begin{bmatrix} \mathbf{S} \times \begin{bmatrix} \mathbf{G}_{x}^{\text{sv1}} \\ \mathbf{G}_{y}^{\text{sv1}} \\ \mathbf{G}_{z}^{\text{sv1}} \\ 1 \end{bmatrix} \mathbf{S} \times \begin{bmatrix} \mathbf{G}_{x}^{\text{sv2}} \\ \mathbf{G}_{y}^{\text{sv2}} \\ \mathbf{G}_{z}^{\text{sv2}} \\ 1 \end{bmatrix} \mathbf{S} \times \begin{bmatrix} \mathbf{G}_{x}^{\text{sv3}} \\ \mathbf{G}_{y}^{\text{sv3}} \\ \mathbf{G}_{z}^{\text{sv3}} \\ 1 \end{bmatrix} \mathbf{S} \times \begin{bmatrix} \mathbf{G}_{x}^{\text{sv4}} \\ \mathbf{G}_{y}^{\text{sv4}} \\ \mathbf{G}_{z}^{\text{sv4}} \\ 1 \end{bmatrix}$$

**STEP 6.** Compute the instantaneous position solution error for the kth solution time.

$$\Delta \overline{\mathbf{x}}(t_k) = \mathbf{G}_{enu}^{-1} \Delta \overline{\mathbf{r}}(t_k) = \mathbf{K} \Delta \overline{\mathbf{r}}(t_k), \text{ or } \begin{bmatrix} \Delta e(t_k) \\ \Delta n(t_k) \\ \Delta u(t_k) \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix} \begin{bmatrix} \Delta r_{sv1}(t_k) \\ \Delta r_{sv2}(t_k) \\ \Delta r_{sv3}(t_k) \\ \Delta r_{sv4}(t_k) \end{bmatrix}$$

where:

 $t_k = k \frac{th}{s}$  solution time corresponding to the signal reception times for the four satellites

 $\Delta \overline{x}(t_k)$  = Position solution error vector in local coordinates (east, north, up and time) at the  $k^{\underline{th}}$  solution time

 $\Delta \overline{\mathbf{r}}(t_k) = \Delta r_{svi}(t_k)$  values from Step 4, for the four satellites used in the k-position solution

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## **SECTION 4.0 Performance Measurement Algorithms**

This section defines the measurement algorithms for evaluating GPS performance against the established performance standards. The standards which will be used to assess measurement results are defined prior to the definition of the associated algorithm. Each algorithm is stated in terms of the steps which must be followed to correctly implement the algorithm. Where neces sary, specific equations are provided to support algorithm implementation.

Note that interpolation is not required between points to determine 95% or 99.99% confidence values. It is expected that less than 0.1% uncertainty will be induced in any confidence value associated with a sample distribution, as long as sample size and rate requirements are met.

The following nomenclature is used in the measurement algorithm definitions.

- MDATA Number of missed data points over the measurement interval (due to equipment failure, etc)
- MCOV Number of points where coverage did not meet standard conditions
- MAVL Number of points where service availability did not meet standard conditions
- MREL Number of points where service reliability did not meet standard conditions
- MRAN<sub>svi</sub> Number of points where the i<sup>th</sup> satellite was unhealthy

## 4.1 Coverage Measurement Algorithm

The standard to be used to evaluate coverage performance is defined below:

Coverage Standard	Conditions and Constraints
C <sub>4SV</sub> ≥ 96.9% at worst-case point	<ul> <li>Probability of 4 or more satellites in view over any 24 hour interval, for the worst-case point on the globe</li> <li>4 satellites must provide PDOP of 6 or less</li> <li>5° mask angle with no obscura</li> <li>Standard is predicated on 24 operational satellites, as the constellation is defined in the almanac</li> </ul>

The coverage performance algorithm is defined in the following steps.

- **STEP 1.** Use the almanac from the current constellation to generate satellite position estimates (including unhealthy satellites) every second over 24 hours.
- STEP 2. Generate elevation angles for each satellite with respect to the desired location.
- STEP 3. If four or more satellites are visible (elevation angle above the mask angle of 5) at the kth time, select four satellites based upon the set which provides the smallest PDOP. If a set is found that provides a PDOP of six or less, the instantaneous coverage flag (C) is equal to one. If less than four satellites are visible or a PDOP of six or less is not supported by any combination of four satellites,  $C_p$  is equal to zero. **STEP 4.** Compute the coverage percentage  $(C_{4\text{SV}})$ , based upon the instantaneous coverage
- values.

$$S_{COV} = 86,400$$

$$\sum_{S_{COV}}^{S_{COV}} C_{p}$$

$$C_{4SV} = \frac{1}{S_{COV}} \times 100\%$$

## 4.2 Service Availability Measurement Algorithm

The standards to be used to evaluate service availability performance are defined below:

Service Availability Standard	Conditions and Constraints
A <sub>AVE</sub> ≥ 99.16% single point average	<ul> <li>Conditioned on coverage standard</li> <li>Standard based on a typical 24 hour interval, for the worst-case point on the globe</li> <li>Typical 24 hour interval defined using averaging period of 30 days</li> </ul>
A <sub>4SV</sub> ≥ 83.92% at worst-case point on worst-case day	<ul> <li>Conditioned on coverage standard</li> <li>Standard based on a worst-case 24 hour interval, for the worst-case point on the globe</li> </ul>

The service availability performance algorithm is defined in the following steps.

- STEP 1. Use receiver at desired location to track (nominally) all satellites in view.
- STEP 2. Check coverage -- determine whether or not coverage standard conditions are satisfied each second over 24 hours. If coverage is not available at a time, service availability is not assessed at that time. The number of points not qualified to support a service availability assessment is defined by the quantity MCOV.

$$MCOV = 86400 - \sum_{p=1}^{S_{COV}} C_p$$

- STEP 3. For each time increment, use the ephemeris to determine whether or not all four satellites are set healthy. If so, then the service availability flag ( $A_p$ ) equals one. If not,  $A_p$  equals zero. If one or more of the satellites providing coverage are unavailable, recompute the optimum selection of four based upon the remaining satellites. If a satellite combination providing a PDOP of six or less is not available,  $A_p$  equals zero.
- **STEP 4.** Compute  $S_{AVL}$ , the total number of valid service availability measurement data points. MDATA equals the number of sample points where measurements were not taken, due to factors such as equipment failure. Compute the daily service availability percentage  $(A_{4SV})$ , based upon the instantaneous service availability values.

$$S_{AVL} = 86400 - MCOV - MDATA$$

$$A_{4SV}(day_d) = \frac{\sum_{p=1}^{S_{AVL}} A_p}{S_{AVL}} \times 100\%$$

**STEP 5.** Compute the average daily service availability percentage (A<sub>AVE</sub>), based upon thirty contiguous days of daily service availability values. Each of the daily values (A<sub>ASVd</sub>) are computed using the four steps defined above.

$$A_{AVE} = \frac{\sum_{d=1}^{30} A_{4SV}(day_d)}{30}$$

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## 4.3 Service Reliability Measurement Algorithm

The standard to be used to evaluate service reliability performance is defined below:

Service Reliability Standard	Conditions and Constraints
R <sub>AVE</sub> ≥ 99.79% single point average	<ul> <li>Conditioned on coverage and service availability standards</li> <li>500 meter NTE predictable horizontal error reliability threshold</li> <li>Standard based on a measurement interval of one year; average of daily values from the worst-case point on the globe</li> <li>Standard predicated on a maximum of 18 hours of major service failure behavior over the measurement interval</li> </ul>

Service reliability is evaluated in terms of the accumulated service failure duration, as defined below.

- STEP 1. Use receiver at desired location to track (nominally) all satellites in view each second over a 24 hour period.
- STEP 2. Check service availability -- determine whether or not service availability standard conditions are satisfied each second. If the service is not available at a time, service reliability is not assessed at that time. The number of points not qualified to support a service reliability assessment is defined by the quantity MAVL.

$$MAVL = 86400 - \sum_{p=1}^{S_{AVL}} A_p$$

- STEP 3. Measure the instantaneous predictable horizontal error, as defined in Section 4.4.1.1, Steps 1-3.
- STEP 4. Determine whether the instantaneous horizontal error exceeds 500 meters. If so, the
- service reliability flag  $(R_p)$  equals one. If not,  $R_p$  equals zero. **STEP 5.** Compute  $S_{REL}$ , the total number of valid service reliability measurement data points. MDATA equals the number of sample points where measurements were not taken, due to such factors as equipment failure. Compute the service reliability over 24 hours.

$$S_{REL} = 86400 - MCOV - MAVL - MDATA$$

$$R(day_d) = 1 - \frac{\sum_{p=1}^{S_{REL}} R_p}{S_{REL}}$$

**STEP 6**. Compute the average daily service reliability (R<sub>AVE</sub>) over a year, based upon 365 days' worth of daily reliability values.

$$R_{AVE} = \frac{\sum_{d=1}^{365} R(day_d)}{365} \times 100\%$$

## 4.4 Accuracy Measurement Algorithms

The accuracy measurement algorithms are segregated into the five different aspects of accuracy: predictable, repeatable, relative, time transfer and range domain accuracy. The standards to be used to evaluate accuracy performance are defined prior to the algorithm for each respective aspect of accuracy.

Once positioning accuracy data is collected over the 24 hour measurement interval, check service reliability and determine whether or not service reliability standard conditions are satisfied each second. If the service is not reliable at a time, positioning accuracy is not assessed at that time. The number of points not qualified to support a positioning accuracy assessment is defined by the quantity MREL. Compute  $S_{ACC}$ , the total number of valid positioning accuracy measurement data points. MDATA equals the number of sample points where measurements were not taken, due to such factors as equipment failure.

$$MREL = \sum_{p=1}^{S_{REL}} R_p$$

$$S_{ACC} = 86400 - MCOV - MAVL - MREL - MDATA$$

Range domain accuracy measurement validation is handled differently than the positioning accuracy methodology just discussed. During the 24 hour range data collection period, determine if each satellite being tracked is healthy for each measurement. If it is not, range domain accuracy is not assessed for that satellite at that time, and the quantity  $H_{svi}$  equals one. If the satellite is healthy,  $H_{svi}$  equals zero. The number of data points not qualified to support a range domain accuracy assessment for the  $\frac{th}{t}$  satellite is defined by the quantity  $MRAN_{svi}$ . Note that MRAN will in general require computation over two or more passes of any given satellite. The variable tn corresponds to the track number for the ith satellite over any arbitrary 24 hour interval. The  $n^{th}$  track number beginning time is denoted by BT(tn) and the end time is denoted by ET(tn). BT and ET are expressed in units of hours, nominally in terms of local or universal time.

$$MRAN_{svi} = \sum_{tn=1}^{n} MRAN_{svi}^{tn}$$

Compute  $S_{RAN}^{svi}$ , the total number of valid range domain accuracy measurement data points for the  $i^{th}$  satellite. MDATA equals the number of sample points where measurements were not taken, due to such factors as equipment failure.  $TT_{svi}$  equals the total accumulated track time for the  $i^{th}$  satellite over the 24 hour interval, expressed in units of seconds.

$$TT_{svi} = INTEGER \left[ \sum_{tn=1}^{n} (ET(tn) - BT(tn)) \times 3,600 \right]$$

$$S_{RAN}^{svi} = TT_{svi} - MRAN_{svi} - MDATA$$

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## 4.4.1 Predictable Accuracy Measurement Algorithm

The standards to be used to evaluate predictable accuracy performance are defined below:

Predictable Accuracy Standard	Conditions and Constraints
$\Delta$ HPRE <sub>95</sub> $\leq$ 100 meters horizontal error 95% of time $\Delta$ UPRE <sub>95</sub> $\leq$ 156 meters vertical error 95% of time $\Delta$ HPRE <sub>99,99</sub> $\leq$ 300 meters horizontal	<ul> <li>Conditioned on coverage, service availability and service reliability standards</li> <li>Standard based on a measurement interval of 24 hours, for any point on the globe</li> </ul>
error 99.99% of time $\Delta \text{UPRE}_{99.99} \le 500 \text{ meters vertical}$	
error 99.99% of time	

#### 4.4.1.1 Horizontal Predictable Accuracy Measurement

The horizontal predictable accuracy performance algorithm is defined in the following steps.

STEP 1. Compute and record instantaneous position solutions every second for 24 hours.

**STEP 2.** Compute the east and north instantaneous errors (in meters) at each time  $t_k$ .

$$\Delta e(t_k) = \left[\lambda_{\text{measured}}(t_k) - \lambda_{\text{site}}\right] 111319.4908 \cos \phi_{\text{site}}$$
 
$$\Delta n(t_k) = \left[\phi_{\text{measured}}(t_k) - \phi_{\text{site}}\right] 111319.4908$$

or alternatively,

$$\begin{split} & \Delta e(t_k) = K_{11} \Delta r_{sv1}(t_k) + K_{21} \Delta r_{sv2}(t_k) + K_{31} \Delta r_{sv3}(t_k) + K_{41} \Delta r_{sv4}(t_k) \\ & \Delta n(t_k) = K_{12} \Delta r_{sv1}(t_k) + K_{22} \Delta r_{sv2}(t_k) + K_{32} \Delta r_{sv3}(t_k) + K_{42} \Delta r_{sv4}(t_k) \end{split}$$

**STEP 3.** Compute the instantaneous horizontal error.

$$\Delta H(t_k) = \left[ \left( \Delta e(t_k) \right)^2 + \left( \Delta n(t_k) \right)^2 \right]^{1/2}$$

STEP 4. Rank order the measurements, and find the  $r^{th}$  sample associated with the 95<sup>th</sup> percentile.  $S_{ACC}$  equals the number of samples over the measurement interval.

$$\Delta$$
HPRE<sub>95</sub> =  $\Delta$ H value at n = INTEGER(0.95 x S<sub>ACC</sub>)

Use the same process (Steps 1-3) to find the nth sample associated with the 99.99 percentile.

$$\Delta$$
HPRE<sub>99.99</sub> =  $\Delta$ H value at n = INTEGER(0.9999 x S<sub>ACC</sub>)

4.4.1.2 Vertical Predictable Accuracy Measurement

The vertical predictable accuracy performance algorithm is defined in the following steps.

STEP 1. Compute instantaneous position solutions every second for 24 hours.

**STEP 2.** Compute the instantaneous vertical error (in meters) at each time t<sub>k</sub>.

$$\Delta u(t_k) = Altitude_{measured}(t_k) - Altitude_{site}$$

or alternatively,

$$\Delta u(t_k) = K_{13} \Delta r_{sv1}(t_k) + K_{23} \Delta r_{sv2}(t_k) + K_{33} \Delta r_{sv3}(t_k) + K_{43} \Delta r_{sv4}(t_k)$$

STEP 3. Take the absolute value of each measurement, rank order the measurements, and find the nth sample associated with the 95th percentile. S<sub>ACC</sub> equals the number of samples over the measurement interval.

$$\Delta UPRE_{95} = \Delta u$$
 value at n = INTEGER(0.95 x S<sub>ACC</sub>)

Use the same process (Steps 1-2) to find the nth sample associated with the 99.99 percentile.

$$\Delta UPRE_{99.99} = \Delta u$$
 value at n = INTEGER(0.9999 x S<sub>ACC</sub>)

## 4.4.2 Repeatable Accuracy Measurement Algorithm

The standards to be used to evaluate repeatable accuracy performance are defined below:

Repeatable Accuracy Standard	Conditions and Constraints
	Conditioned on coverage, service availability and
error 95% of time	service reliability standards
ΔUREP <sub>95</sub> ≤ 221 meters vertical error	<ul> <li>Standard based on a measurement interval of 24</li> </ul>
95% of time	hours, for any point on the globe

#### 4.4.2.1 Horizontal Repeatable Accuracy Measurement

The horizontal repeatable accuracy performance algorithm is defined in the following steps.

- **STEP 1.** Measure and record position at time  $t_k$ . Repeat each second for 24 hours plus  $\Delta t$ .
- **STEP 2.** Measure position at time  $t_k+\Delta t$ .  $\Delta t$  must be longer than the position error correlation time constant. 15 minutes is recommended as the minimum size.
- **STEP 3.** Compute the difference in position between the two times. If measurements are taken from a surveyed benchmark, use the error vectors taken at the two times.

$$\Delta \overline{P}_{\text{repeat}} \big( t_k + \Delta t \big) = \Delta \overline{P} \big( t_k + \Delta t \big) - \Delta \overline{P} \big( t_k \big) \,, \, \text{where the east and north components are:}$$

$$\textbf{EAST:} \qquad \Delta e_{\text{repeat}} \left( t_{\text{k}} \ + \Delta t \right) = \Delta e \left( t_{\text{k}} \ + \Delta t \right) - \Delta e \left( t_{\text{k}} \right)$$

**NORTH:** 
$$\Delta n_{\text{repeat}}(t_k + \Delta t) = \Delta n(t_k + \Delta t) - \Delta n(t_k)$$

If measurements are taken without a survey, directly compute the repeatable error vector and convert the east and north errors from an angular to a linear quantity.

$$\Delta \overline{P}_{repeat}(t_k + \Delta t) = \overline{P}(t_k + \Delta t) - \overline{P}(t_k)$$
, where the east and north components are:

$$\Delta \varphi = \varphi_{\text{measured}} \left( t_k \, + \Delta t \right) - \varphi_{\text{measured}} \left( t_k \, \right) \text{; difference in latitude measurements}$$

$$\Delta \lambda = \lambda_{\text{measured}} \left( t_k \right. \\ \left. + \Delta t \right) - \lambda_{\text{measured}} \left( t_k \right) \text{; difference in longitude measurements}$$

$$\textbf{EAST:} \qquad \Delta e_{\text{repeat}} \left( t_k + \Delta t \right) = \left[ \Delta \lambda \right] 111319.4908 \cos \left[ \phi_{\text{measured}} \left( t_k + \Delta t \right) - \frac{\Delta \phi}{2} \right]$$

**NORTH:** 
$$\Delta n_{repeat} (t_k + \Delta t) = \Delta \phi *111319.4908$$

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**STEP 4.** Compute the difference in horizontal position magnitude at  $t_k + \Delta t$ .

$$\Delta H_{repeat} \left( t_k + \Delta t \right) = \left[ \left( \Delta e_{repeat} \left( t_k + \Delta t \right) \right)^2 + \left( \Delta n_{repeat} \left( t_k + \Delta t \right) \right)^2 \right]^{\frac{1}{2}}$$

STEP 5. Rank order the measurements, and find the nth sample associated with the 95th percentile. S<sub>ACC</sub> equals the number of samples over the measurement interval.

$$\Delta$$
HREP<sub>95</sub> =  $\Delta$ H<sub>repeat</sub> value at n = INTEGER(0.95 x S<sub>ACC</sub>)

4.4.2.2 Vertical Repeatable Error Measurement

The vertical repeatable accuracy performance algorithm is defined in the following steps.

- **STEP 1**. Repeat steps 1 and 2 from the horizontal repeatable position error computation Section 4.4.2.1).
- **STEP 2.** Compute the difference in altitude between the two times.

$$\Delta u_{repeat}(t_k + \Delta t) = \Delta u(t_k + \Delta t) - \Delta u(t_k)$$

**STEP 3.** Take the absolute value of each measurement, rank order the measurements, and find the  $n^{\underline{th}}$  sample associated with the  $95^{\underline{th}}$  percentile.  $S_{ACC}$  equals the number of samples over the measurement interval.

$$\Delta UREP_{95} = \Delta u_{repeat}$$
 value at n = INTEGER(0.95 x S<sub>ACC</sub>)

## 4.4.3 Relative Accuracy Measurement Algorithm

The standards to be used to evaluate relative accuracy performance are defined below:

Relative Accuracy Standard	Conditions and Constraints
$\Delta$ HREL $_{95}$ ≤ 1.0 meters horizontal error 95% of time $\Delta$ UREL $_{95}$ ≤ 1.5 meters vertical error 95% of time	<ul> <li>Conditioned on coverage, service availability and service reliability standards</li> <li>Standard based on a measurement interval of 24 hours, for any point on the globe</li> <li>Standard presumes that the receivers base their position solutions on the same satellites, with position solutions computed at approximately the same time</li> </ul>

Note that the relative accuracy standards are based upon signal-in-space errors, and do not in clude the receiver ranging measurement contribution to positioning error. Users who measure relative accuracy can expect to experience 6 meters horizontal error (95%) and 9 meters vertical error (95%), due to the contribution of the two receivers to the relative position solution error.

4.4.3.1 Horizontal Relative Accuracy Measurement

The horizontal relative accuracy performance algorithm is defined in the following steps.

- **STEP 1.** Gather and record ranging measurements and navigation message data from two receivers for all satellites in view each second for 24 hours. The receivers must be colocated or sitting at surveyed locations (recommended to be within 40 kilometers of one another, to support consistent satellite tracking coordination between the two receivers).
- **STEP 2.** Use the ranging measurements and navigation data to generate range residuals for both sets of measurements.

- **STEP 3.** Use the range residuals to generate (in post-processing) position residual estimates for each measurement set. The satellite selection strategy must coordinate satellite selections between the two receivers. The position solutions from both receivers must meet coverage, service availability and service reliability conditions.
- STEP 4. Compute the instantaneous predictable error for the two receivers. Rotate the resulting ECEF vectors into the local coordinates of either of the two receivers. Compute the relative horizontal error components.

$$\Delta \mathbf{e}_{\text{rel}}(\mathbf{t}_{k}) = \Delta \mathbf{e}_{\text{receiver1}}(\mathbf{t}_{k}) - \Delta \mathbf{e}_{\text{receiver2}}(\mathbf{t}_{k})$$
$$\Delta \mathbf{n}_{\text{rel}}(\mathbf{t}_{k}) = \Delta \mathbf{n}_{\text{receiver1}}(\mathbf{t}_{k}) - \Delta \mathbf{n}_{\text{receiver2}}(\mathbf{t}_{k})$$

$$\Delta H_{\text{rel}}\!\left(t_{k}\right)\!=\!\left\lceil\left(\Delta e_{\text{rel}}\!\left(t_{k}\right)\right)^{2}+\left(\Delta n_{\text{rel}}\!\left(t_{k}\right)\right)^{2}\right\rceil^{1/2}$$

**STEP 5.** Rank order the measurements, and find the  $r^{th}$  sample associated with the 95<sup>th</sup> percentile. S<sub>ACC</sub> equals the number of samples over the measurement interval.

$$\Delta$$
HREL<sub>95</sub> =  $\Delta$ H<sub>rel</sub> value at n = INTEGER(0.95 x S<sub>ACC</sub>)

4.4.3.2 Vertical Relative Accuracy Measurement

The vertical relative accuracy performance algorithm is defined in the following steps.

- STEP 1. Perform Steps 1-3 in the horizontal relative accuracy methodology (Section 4.3.3.1).
- **STEP 2.** Compute the instantaneous relative vector between the two receivers. Rotate the re sulting ECEF vector into the local coordinates of either of the two receivers.

$$\Delta u_{rel}(t_k) = \Delta u_{receiver1}(t_k) - \Delta u_{receiver2}(t_k)$$

**STEP 3.** Take the absolute value of each measurement, rank order the measurements, and find the  $n^{\underline{th}}$  sample associated with the  $95^{\underline{th}}$  percentile.  $S_{ACC}$  equals the number of samples over the measurement interval.

$$\Delta UREL_{95} = \Delta u_{rel}$$
 value at n = INTEGER(0.95 x S<sub>ACC</sub>)

4.4.3.3 A Note on Relative Accuracy with Uncoordinated Satellite Tracking

Note that in the event that measurements are taken with no attempt to coordinate satellite track ing, accuracy will vary widely as a function of distance between receivers and the number of sat ellites which their solutions have in common. Accuracy can be as good as that provided by the correlated solution, and as bad as repeatable error over long sample intervals from completely uncorrelated solutions. Receivers relatively close together will tend to select common satellites the majority of the time, so accuracy should tend towards the completely correlated values.

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## 4.4.4 Time Transfer Accuracy Measurement Algorithm

The standard to be used to evaluate time transfer accuracy performance is defined below:

Time Transfer Accuracy Standard	Conditions and Constraints
$\Delta t_{u95} \leq 340$ nanoseconds time transfer error 95% of time	<ul> <li>Conditioned on coverage, service availability and service reliability standards</li> <li>Standard based upon SPS receiver time as computed using the output of the position solution</li> <li>Standard based on a sample interval of 24 hours, for any point on the globe</li> <li>Standard is defined with respect to Universal Coordinated Time, as it is maintained by the United States Naval Observatory</li> </ul>

The time transfer accuracy performance algorithm is defined in the following steps.

- **STEP 1.** Measure instantaneous range residuals for the satellites selected for the optimum position solution. Range residual time tags must be based upon a measurement system timing system tied to USNO UTC to within 10 ns Root Mean Square (RMS). Use the residuals in the linearized navigation equations to generate and record a solution each minute over 24 hours. The position solution must meet coverage, service availability and service reliability conditions.
- **STEP 2.** From the position solution equations, estimate the receiver's time offset with respect to GPS time.

$$\Delta t_u(t_k) = \sum_{i=1}^4 K_{4i} \, \frac{\Delta r_{svi}(t_k)}{c} \, , \, \text{where c = WGS-84 value for speed-of-light in a vacuum}$$

STEP 3. Apply the UTC correction from the navigation message to the GPS time offset estimate.
 STEP 4. Take the absolute value of each measurement, rank order the measurements, and find the nth sample associated with the 95th percentile. S<sub>ACC</sub> equals the number of samples over the measurement interval.

$$\Delta t_{u95} = \Delta t_u$$
 value at n = INTEGER(0.95 x S<sub>ACC</sub>)

#### 4.4.5 Range Domain Accuracy Measurement Algorithm

The range domain accuracy algorithm supports the evaluation of four different aspects of range domain accuracy for each satellite: maximum range error, maximum range rate error, the 95% confidence value of range acceleration error, and maximum range acceleration error. The standard to be used to evaluate range domain accuracy performance is defined below:

Range Domain Accuracy Standard	Conditions and Constraints
RE <sup>max</sup> <sub>svi</sub> ≤ 150 meters	Conditioned on satellite indicating healthy status
RR <sub>svi</sub> ≤ 2 meters/second	Standard based on a measurement interval of 24 hours, for any point on the globe
RA <sub>svi</sub> ≤ 8 millimeters/second <sup>2</sup>	Standard restricted to range domain errors
RA <sub>svi</sub> ≤ 19 millimeters/second <sup>2</sup>	allocated to space/control segments
341	Standards are not constellation values each satellite is required to meet the standards
	Assessment requires minimum of four hours of da over the 24 hour period for any given satellite in
	order to evaluate that satellite against the standard

The range domain accuracy performance algorithm is defined in the following steps.

- STEP 1. Synchronize the measurement system with GPS time, by computing the instantaneous time bias of the measurement system with respect to GPS time (T<sub>gpsbias</sub>). This must be accomplished with a time scale error of no more than 10 nanoseconds RMS time error with respect to GPS time.
- **STEP 2.** For each time  $t_k$ , apply the time bias correction determined in Step 1 ( $T_{gpsbias}$ ) to each range residual to generate instantaneous range error values ( $RE_{svi}$ ) for each satellite.

$$RE_{svi}(t_k) = \Delta r_{svi}(t_k) - T_{gpsbias}(t_k)$$

STEP 3. Compute the instantaneous range rate error (RR $_{svi}$ ) based upon successive range error values. To minimize multipath and tropospheric effects on the measurement, apply a  $10^{\circ}$  mask angle. Apply the time rate correction (TR $_{gpsbias}$ ) to each range rate error value.

$$RR_{svi}(t_k) = \frac{RE_{svi}(t_k) - RE_{svi}(t_{k-1})}{t_k - t_{k-1}} - TR_{gpsbias}$$

NOTE: If the user is not interested in range error except as an intermediate step to computing range rate and range acceleration errors, the only requirement is to maintain the time scale rate with respect to GPS time such that it does not contribute more than 10 millimeters/second to the range rate error measurement.

**STEP 4.** Compute the instantaneous range acceleration error (RA<sub>svi</sub>) based upon successive range rate error values.

$$RA_{svi}(t_k) = \frac{RR_{svi}(t_k) - RR_{svi}(t_{k-1})}{t_k - t_{k-1}}$$

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STEP 5. Filter the range rate values to mitigate receiver noise and multipath effects. The filter algorithm used must impose minimal distortion on the measured range obmain error waveform. Filter response up to a frequency of approximately 2 Hertz must be very close to 0 dB.

NOTE: This filter step may not be necessary if a low noise receiver is used. Low noise is defined here to be a receiver contribution of no more than 1 centimeter RMS noise to each pseudo range measurement.

NOTE: The use of two frequency receivers that make use of carrier tracking and cross-correlation technologies is allowed for this measurement process. However, the user must ensure that cycle slips do not corrupt measurements.

**STEP 6.** Smooth the range acceleration error values to mitigate receiver noise and multipath effects. We recommend a smoothing interval of 15 seconds.

$$RA_{svi}^{smooth}(t_k) = \frac{\sum_{p=k-14}^{k} RA_{svi}(t_p)}{15}$$

- STEP 7. Store instantaneous range error, range rate error and range acceleration error data each second over a 24 hour interval for each satellite in view.
- Take the absolute value of each range error measurement for the statellite, and sort to determine the nth sample associated with the 100th percentile (maximum magnitude). Repeat for each satellite. S<sub>RAN</sub> equals the number of valid measurements over the measurement interval. Note that if S<sub>RAN</sub> is less than 14,400, that satellite may not be assessed over that 24 hour interval for any of the range domain accuracy parameters.

$$RE_{svi}^{max} = RE_{svi}$$
 value at  $n = S_{RAN}^{svi}$ 

STEP 9. Take the absolute value of each range rate error measurement for the the satellite, and sort to determine the nthe sample associated with the 100th percentile (maximum magnitude). Repeat for each satellite.

$$RR_{svi}^{max} = RR_{svi}$$
 value at  $n = S_{RAN}^{svi}$ 

**STEP 10.** Take the absolute value of each range acceleration error measurement for the  $^{th}$  satellite, and sort to determine the  $n^{th}$  samples associated with the  $95^{th}$  and  $100^{th}$  percentiles. Repeat for each satellite.

$$\begin{split} RA_{svi}^{95\%} &= RR_{svi} \text{ value at } n = INTEGER \Big( 0.95 \text{ x } S_{RAN}^{svi} \Big) \\ RA_{svi}^{max} &= RR_{svi} \text{ value at } n = S_{RAN}^{svi} \end{split}$$

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