SECTION III

METHODS FOR PRECISE FREQUENCY STANDARD INTERCOMPARISON

GENERAL

This section presents methods by which frequency differences between two frequency sources can be determined. We assume for these methods that the two frequency sources are physically near each other. External references such as VLF, LF, LORAN C etc. will be discussed in the following section. This section establishes concepts and measurements fundamental to the time-transfer techniques of the next section.

To be useful, methods for comparing frequency sources, especially precision frequency sources, must be capable of resolving extremely small differences. This section describes eight such useful methods that have varying degrees of complexity and resolution. Two methods are described which involve the use of an oscilloscope for interpretation, while two others use the period measuring capability of an electronic counter for comparison purposes. Three other methods are presented which involve the direct measurement of phase characteristics versus-time. The last technique derives the frequency from measurements of time. Among these methods, at least one or two should be suitable for any type of short or long-term frequency standard comparison desired.

OSCILLOSCOPE LISSAJOUS PATTERNS

A well known method of comparing two frequencies is to observe the pattern displayed on an oscilloscope when one frequency is applied to its horizontal input and the other to its vertical input. If the ratio of the frequencies is an integer or the ratio of two integers, the resulting pattern, called a Lissajous figure (Figure 3-1) can be interpreted to determine this frequency ratio. If a rectangle is imagined to bound the pattern, the number of points where the loops are tangent on one vertical and one adjacent horizontal side directly indicates the ratio of the two frequencies.

When the two frequencies being compared in the Lissajous display are quite similar (within a few hertz) the trace will be elliptical. Slight frequency differences cause the ellipse to roll repeatedly through all orientations from 0° to 360°. It is possible to time the completion of a 360° sequence

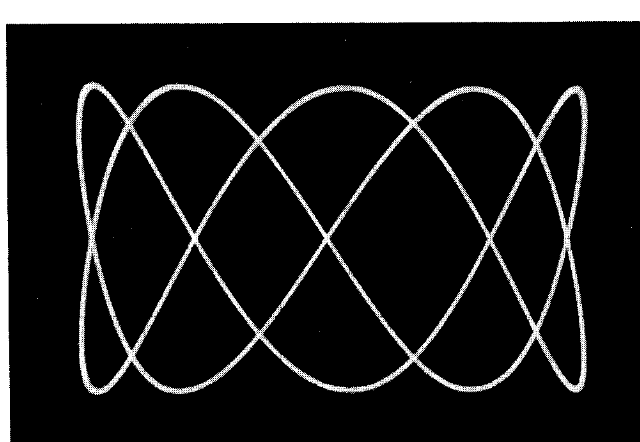

Figure 3-1. Lissajous Pattern

and to find the frequency difference, which is the reciprocal of this time (in seconds). To match the frequency of an oscillator closely to that of a house frequency standard, the oscillator is adjusted until the ellipse is stationary.

The practical limit for use of this technique for frequency offset adjustments and comparisons is about 1 part in 10^9. It is of limited practicality for quantitative fractional frequency deviation measurements. Reference to a comprehensive presentation of Lissajous patterns is advisable before attempting to use them for comparison purposes.*

OSCILLOSCOPE PATTERN DRIFT

An oscillator can be compared against a house frequency standard by externally triggering the oscilloscope from the standard while a pattern of several cycles of the oscillator is displayed. The ratio of drift of the oscilloscope pattern is related to the frequency error of the oscillator under test. For example, suppose an HP Model 181A Oscilloscope is being used to check the time base oscillator frequency of an HP Model 5345A Electronic Counter against a house standard such as the HP Model 5065A Rubidium Vapor Frequency Standard. The equipment configuration for this measurement is shown in Figure 3-2.

If the oscilloscope pattern moves to the right, the counter's time base oscillator frequency is low compared to that of the standard; if the pattern moves to the left, the counter frequency is high.

Rate of movement can be interpreted in terms of frequency error in this way: With a 1 MHz signal from the standard used to trigger the display of a 10 MHz signal from the counter's internal oscillator, the time required for the pattern to apparently drift the width of one cycle of the display is noted. Suppose that the pattern drifts left the width of one cycle in a time of 10s; this is equivalent to a frequency difference of 0.1 cycle per second. Frequency error, then is 1 part in 10^8 (high):

$$\frac{\Delta f}{f} = \frac{0.1}{10^7} = 1 \times 10^{-8}$$

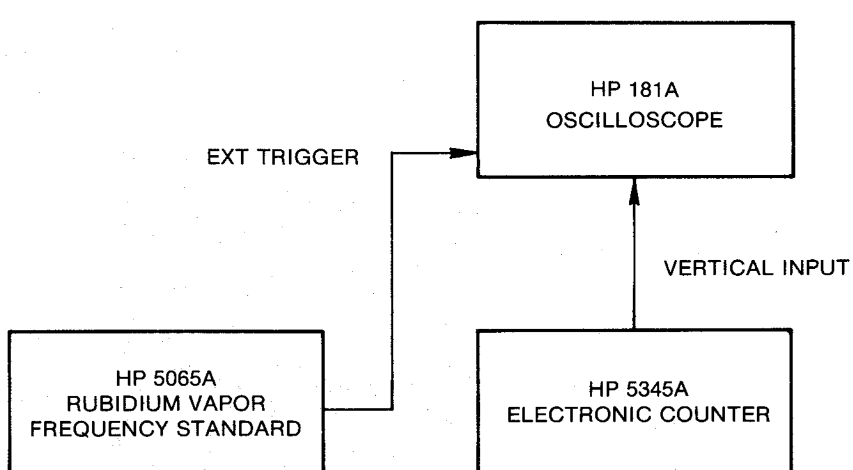

Figure 3-2. Oscilloscope Pattern Drift

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* J. Czech, "The Cathode Ray Oscilloscope" Clever-Hume Press, Ltd., London 1957. See also: Rider and Usland, "Encyclopedia on Cathode Ray Oscilloscopes", Rider, Inc., New York, 1959.

If it takes 100 seconds for the pattern to drift the width of one cycle, the error is 1 part in 10^9.

$$\frac{\Delta f}{f} = \frac{0.01}{10^7} = 1\text{x}10^{-9}$$

If two 100 kHz signals are being compared and movement the width of one cycle takes 1 s, the frequency error is 1 part in 10^5,

$$\frac{\Delta f}{f} = \frac{1}{100\text{x}10^3} = 1\text{x}10^{-5}$$

Each of the above measurements can be made more quickly, but with less accuracy, by letting the pattern drift the width of $^1/_2$ cycle on the oscilloscope. Since many oscilloscopes have a calibrated display there is a temptation to take advantage of this in trying to shorten the observation time still further. Caution should be exercised, however, as inaccuracies in calibration and non-linearity in the display of the oscilloscope can distort the frequency error.

With the oscilloscope pattern drift method, as in the Lissajous method, the largest error will come from the method used for timing. A recommended technique for both of these frequency difference measurements is to adjust the oscillator being tested until there is no apparent movement on the oscilloscope. The resolution of this zeroing adjustment can be extended by orders of magnitude by setting the oscilloscope vertical gain to maximum and increasing the horizontal sweep speed until a single sloping portion of the waveform is visible on the screen. The frequency adjustment being made can be used to slow the phase so that the scope trace remains on screen while raising the horizontal sweep speed. A method of determining $\Delta f/f$ directly, quickly, and with greater accuracy is discussed below.

DIRECT FREQUENCY COMPARISON WITH A COUNTER

This section assumes the reader is familiar with the use and operation of a basic frequency counter whose measurement is cycles sensed per unit time. The details of basic counter measurements are covered in the Operating Manual for such instruments as the 5328A and in Hewlett-Packard AN 172, The Fundamentals of Electronic Frequency Counters.

The frequency counter is an instrument which can be used to measure the frequency of an oscillator with reasonable accuracy. A direct frequency measurement by a counter is in essence a comparison of two oscillators, the oscillator being measured and either the internal oscillator of the counter or an external frequency reference.

The HP 5345A Electronic Counter can be used to accurately compare two oscillators. As with any counter the accuracy of the measurement depends upon the error involved. Counter error consists of three parts:

a. ± Least Significant Digit (LSD) count;
b. ± Time base accuracy (accuracy of interval or external oscillator; and
c. ± Trigger error.

For the 5345A the LSD error is ±1 count if the most significant digit (MSD) is 1 through 4, and ± 2 counts if the MSD is 5 through 9. Unlike conventional counters, the ± 5345A LSD count error relates to the 500 MHz time count register rather than in terms of one cycle of the input frequency. Therefore, with a 100 kHz input and a GATE TIME of 1 second, this error is only 1 x 10^{-8}; furthermore, with the 5345A, this ± count error remains constant for any input frequency for a fixed GATE TIME.

Trigger error is associated with the input signal and changes with waveform and measurement time. Trigger error in the 5345A is basically the same as a conventional counter measuring period or multiple period average and is specified as:

$$\frac{<0.3\%}{\text{number of periods averaged}}$$

for a 10 mVrms input sine wave signal with sine wave noise more than 40 dB below signal level.

For a 5 MHz signal with a 1 second GATE TIME the trigger error is less than $\pm 6 \times 10^{-10}$. Trigger error can be reduced by either increasing the amplitude of the input signal above the 10 mV rms minimum or by measuring a fast rise-time signal if available.

Error in the crystal oscillator "clock" frequency directly influences the accuracy of a measurement as it does for a conventional counter. The standard 5345A uses the HP high stability 10544A 10 MHz crystal oscillator which has an aging rate of $\pm 5 \times 10^{-10}$/day and very good short-term stability. This oscillator satisfies most applications requiring the highest internal time base stability available in any counter manufactured.

The 5345A counter contains circuitry to lock the internal oscillator to any externally connected frequency standard at a subharmonic of 10 MHz in the range of 1 MHz to 10 MHz to give the same time base accuracy as the reference standard.

If a HP 5061A Cesium Beam Frequency Standard is used as the reference standard, the 5345A can be used to make very accurate frequency comparisons with convenient measurement times. If we assume worst case on the 5061A accuracy, the reference standard error will be less than $\pm 1 \times 10^{-11}$ of the international definition of frequency. Let us now measure a 5 MHz signal of 10 mV minimum using a 1 second, 10 second, 100 second, and 1000 second gate time. The equipment setup is as shown in Figure 3-3. For a 1 second gate time, the maximum error would be

$$\begin{aligned}\text{Error Max} &= \pm\text{count} \pm\text{trigger error} \pm\text{oscillator error}\\ &= \pm 2 \text{ counts} \pm \frac{0.3\%}{5 \times 10^6} \pm 1 \times 10^{-11}\\ &= \pm 4 \times 10^{-9} \pm 6 \times 10^{-10} \pm 1 \times 10^{-11}\\ &= \pm 4.61 \times 10^{-9}\end{aligned}$$

For a 10 second gate time

$$\begin{aligned}\text{Error Max} &= \pm 2 \text{ counts} \pm \frac{0.3\%}{5 \times 10^7} \pm 1 \times 10^{-11}\\ &= \pm 4.7 \times 10^{-10}\end{aligned}$$

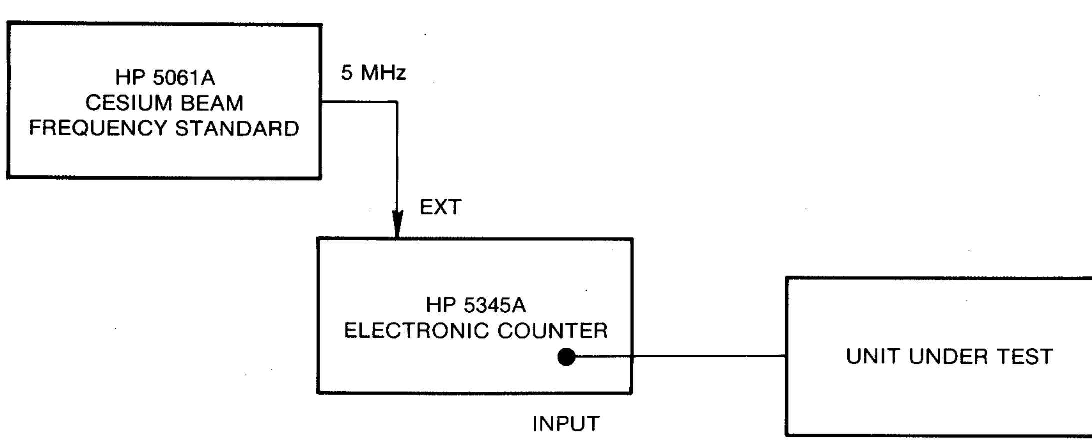


Figure 3-3. Direct Frequency Comparison with a Counter

For a 100 second gate time

$$\text{Error Max} = \pm 2 \text{ counts } \pm \frac{0.3\%}{5 \times 10^8} \pm 1 \times 10^{-11}$$

$$= \pm 4 \times 10^{-11} \pm 6 \times 10^{-12} \pm 1 \times 10^{-11}$$

$$= \pm 5.6 \times 10^{-11}$$

For a 1000 second gate time

$$\text{Error Max} = \pm 4 \times 10^{-12} \pm 6 \times 10^{-13} \pm 1 \times 10^{-11}$$

$$= \pm 1.46 \times 10^{-11}$$

DIRECT COUNTER READOUT OF NORMALIZED OR FRACTIONAL FREQUENCY ERROR

Where a number of comparisons of precision oscillators expected to agree in frequency within parts in 10^6 (or better) is to be made against the reference frequency standard, it is convenient to arrange equipment so that a counter's readout can be interpreted directly in terms of frequency error to an accuracy of parts in 10^6, 10^7, 10^8, etc. The equipment arrangement to accomplish this direct readout includes a reference oscillator offset from the reference standard by a predetermined amount.

The offset frequency from the reference oscillator and the frequency from the oscillator under test are mixed and the period of their difference frequency is measured. Such a comparison constitutes a short-term stability measurement; the changes in period of the difference frequency indicate the instabilities of the test oscillator and the measuring system including the reference frequency. The period displayed on the counter's readout can easily be interpreted (digit by digit from left to right) as frequency error (for example, parts in 10^6, 10^7. etc.).

To illustrate this method, assume a quartz oscillator's 1 MHz output is to be compared with the output of an offset oscillator. The system shown in Figure 3-4 is set up and calibrated by momentarily substituting a known 1 MHz signal (from the house standard) for the output of the oscillator under test. The HP Model 105A/B Quartz Oscillator serving as an offset standard is adjusted in frequency until the counter display is exactly 1,000,000.0 μs, the period of 1 Hz.

The 105A/B Oscillator is now at a frequency of either 1,000,001.0 Hz or 999,999.0 Hz; but more important it is 1 Hz from the known 1 MHz signal. The known signal is now removed from the mixer input and the output of the 1 MHz oscillator under test is applied in its place. If the test oscillator is at the same frequency as the 1 MHz calibration signal the counter will again read 1,000,000.0 μs. If it does not, its error can read directly to parts in 10^X as illustrated in the following example. If the counter's BCD output is converted to analog form and plotted on a strip chart recorder, a plot of $\Delta f/f$ with respect to time is obtained.

As an example, to compute what fractional frequency deviation is indicated for the oscillator under test when the period displayed on the counter's readout is observed to change by one microsecond, the following approximate method can be used:

$$f = \frac{1}{\tau} = \tau^{-1}$$

Where:
f = frequency, Hz
τ = period, s

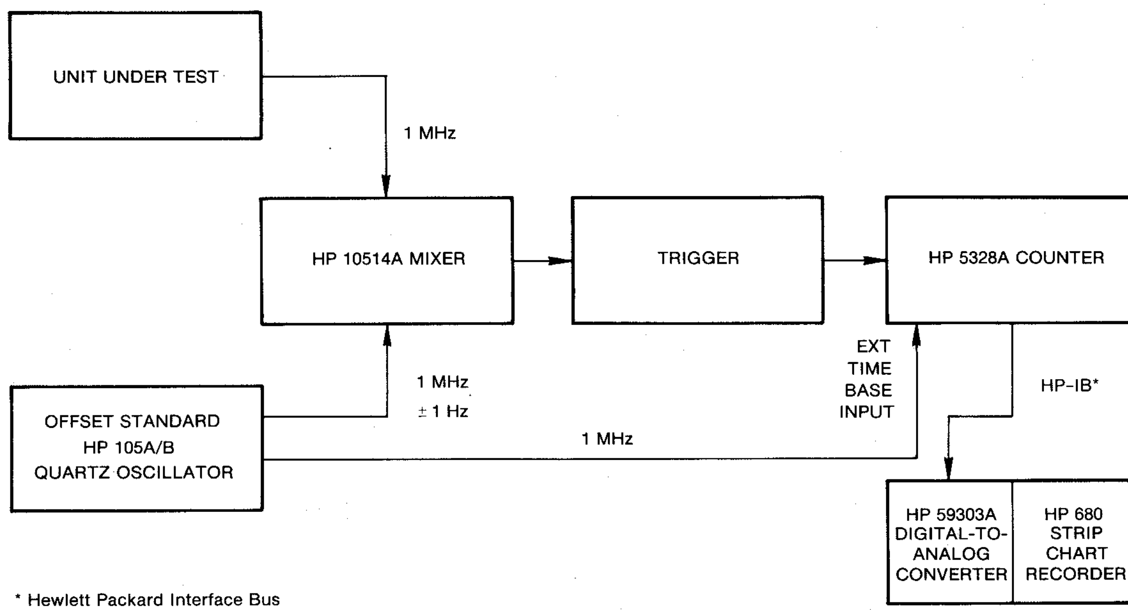


Figure 3-4. Direct Counter Readout of Fractional Frequency Error

Differentiating:

$$df = -\tau^{-2}\,d\tau = -\frac{d\tau}{\tau^2}$$

Dividing both sides by frequency:

$$\frac{df}{f} = -\frac{d\tau}{\tau^2 f}$$

Which for small changes can be approximated as:

$$\frac{\Delta f}{f} = -\frac{\Delta\tau}{\tau^2 f}$$

With the following interpretation:

$\frac{\Delta f}{f}$ = Fractional frequency offset in the frequency being checked

$\Delta\tau$ = Change in the period displayed on the counter

τ = Period of input signal to the counter (difference frequency)

f = Frequency being checked

With the substitution of values:

$\Delta\tau = 1\mu s = 10^{-6}s$

$f = 1\text{ MHz} = 10^6\text{ Hz}$

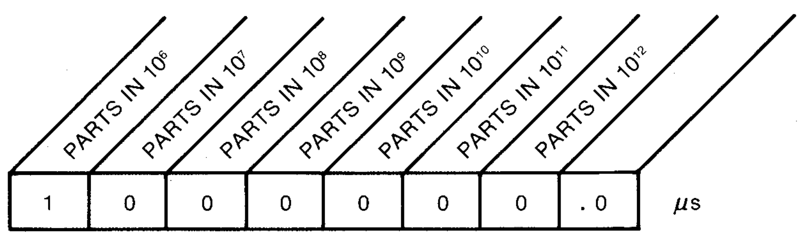
$\tau = 1\text{ s}$

$$\frac{\Delta f}{f} = -\frac{10^{-6}}{\tau^2(10^6)} = -\frac{10^{-6}}{(1)^2 10^6} = -10^{-12}$$

Thus, when the period displayed on the counter is observed to change by 1μs, the fractional frequency deviation is indicated to be one part in 10^{12}. This $\Delta f/f$ is attributed to the oscillator under test. The reference oscillator (and the system) are assumed to be ideal.

Note that this development has neglected measurement system jitter and counter resolution. Period measurements are subject to trigger errors even if the mixing frequency and the counter frequency are known exactly. Although resolution is on the order of 1 part in 10^{12}, absolute accuracy to this level would require great care and is more readily accomplished with other methods.

Using the figures from the above example, the period displayed on the counter's readout can be interpreted in terms of fractional frequency deviation as follows:



By use of this same technique it may be desirable to compare two 5 MHz frequencies. Using the equations previously developed, the period (inverse of the offset frequency) that should be measured is easily determined.

$$\frac{\Delta f}{f} = \frac{\Delta \tau}{\tau^2 f}$$

Solving for τ:

$$\tau = \sqrt{\frac{\Delta \tau}{f\left(\frac{\Delta f}{f}\right)}}$$

If the desired measurement is 1 part in 10^{12} for a 1 μs change in the period measurement display column, then:

$$\frac{\Delta f}{f} = 1\times10^{-12}$$

$$\Delta \tau = 10^{-6}\ s$$

$$f = 5\times10^{6}\ Hz$$

Substituting:

$$\tau = \sqrt{\frac{10^{-6}}{(10^{-12})\,(5\times10^{6})}} = \sqrt{\frac{1}{5}}$$

$$= 0.4472135\ s$$

The offset frequency determined from the desired offset period measurement would be (approximately) 2.25 Hz in order to have a 1 μs change equivalent to a $\Delta f/f$ of 1 part in 10^{12}.

FREQUENCY COMPARISON WITH A VECTOR VOLTMETER

Another method of rapid, accurate frequency comparison involves the use of a vector voltmeter to measure the phase shift versus time between two signal sources. With this method, frequency differences as small as 1 part in 10^{13} can be measured in a few minutes.

Figure 3-5 shows the HP Model 8405A Vector Voltmeter used in a system to compare and record the frequency difference between an HP 5061A Cesium Beam Frequency Standard and an HP 105A/B Quartz Oscillator under test. The phase of the signal from the reference 5061 is compared to the phase of the oscillator's signal and indicated on the 8405A's phasemeter. A dc recorder jack on the rear panel of the 8405A provides a voltage output proportional to the phasemeter reading.*

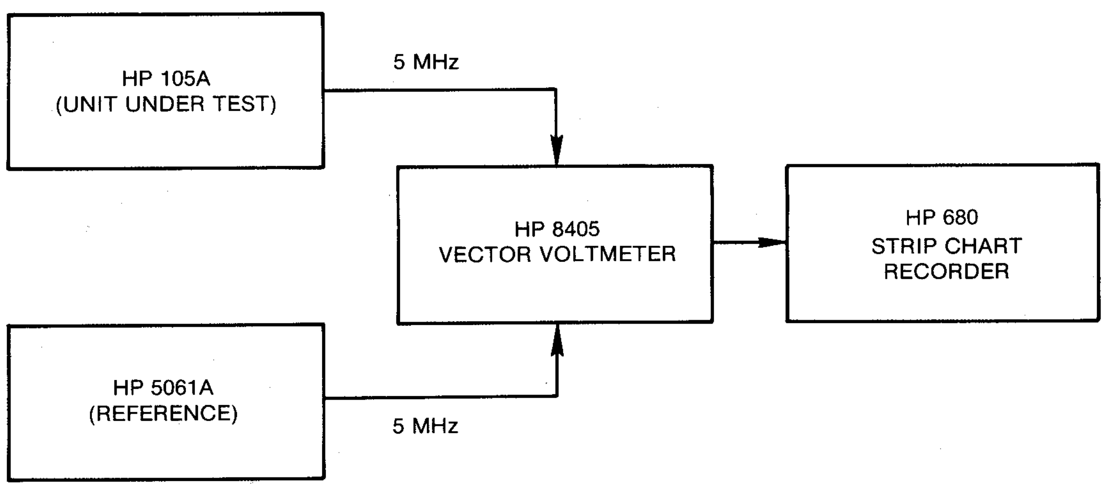


Figure 3-5. Frequency Comparison with a Vector Voltmeter

The phase change and direction can be recorded over a known period of time using a Model 680 Strip Chart Recorder as shown in Figure 3-5. Frequency difference, in proportional parts, can then be calculated and read from the calibrated recorder's output.

The phase angle between two signal sources changes by 360° every second for each cycle (Hz) of their difference in frequency. Stated in another form:

$$\frac{360°/s}{1\ Hz} = \frac{\Delta\phi/t}{\Delta f}$$

Solving for Δf,

$$\Delta f = \frac{\Delta\phi}{360\ t}$$

Dividing by f,

$$\frac{\Delta f}{f} = \frac{\Delta\phi}{360\ tf}$$

Where:

Δf = frequency difference between the two signal sources, Hz.
f = frequency of the standard source, Hz.
$\Delta\phi$ = phase change in degrees during the measurement time.
t = time, in seconds, during which $\Delta\phi$ was measured.
$\frac{\Delta f}{f}$ = fractional frequency offset of the source being checked, dimensionless.

* HP Application Note 77-2, "Precision Frequency Comparison."

Example Measurement and Calculation for Setup in Figure 3-5:

f = 5 MHz, frequency at which phase changes are measured.
$\Delta\phi$ measured with 8405A = 1.3°
t = 60 seconds

$$\frac{\Delta f}{f} = \frac{1.3°}{360° \, (60 \text{ s}) \, (5\text{x}10^6 \text{ Hz})}$$

$$= \frac{1.3}{1.08 \times 10^{11}} = 1.2\text{x}10^{-11}$$

or 1.2 parts in 10^{11}.

During the measurement in this example, the 1.3° phase change observed was the result of a clockwise movement of the phasemeter's indicator. This is interpreted as showing that the test oscillator's signal on the "Channel B" input of the 8405A was leading the standard's signal on the "Channel A" input. Therefore, the test oscillator's frequency was higher than the standard's frequency by 1.2 parts in 10^{11}.

In general, the frequency difference between two oscillators is determined from the slope of phase change versus time. Therefore the phase slope at any time will be proportional to the instantaneous frequency difference between the two oscillators. A constant frequency offset appears as a linear phase versus time trace. A linear drift in frequency results in a parabolic trace.

FREQUENCY COMPARISON WITH A PHASE COMPARATOR AND STRIP CHART RECORDER

Long term frequency comparison of two signal sources can be accomplished quite simply using the system shown in Figure 3-6. The primary component in this system is the HP Model K05-5060A Linear Phase Comparator.

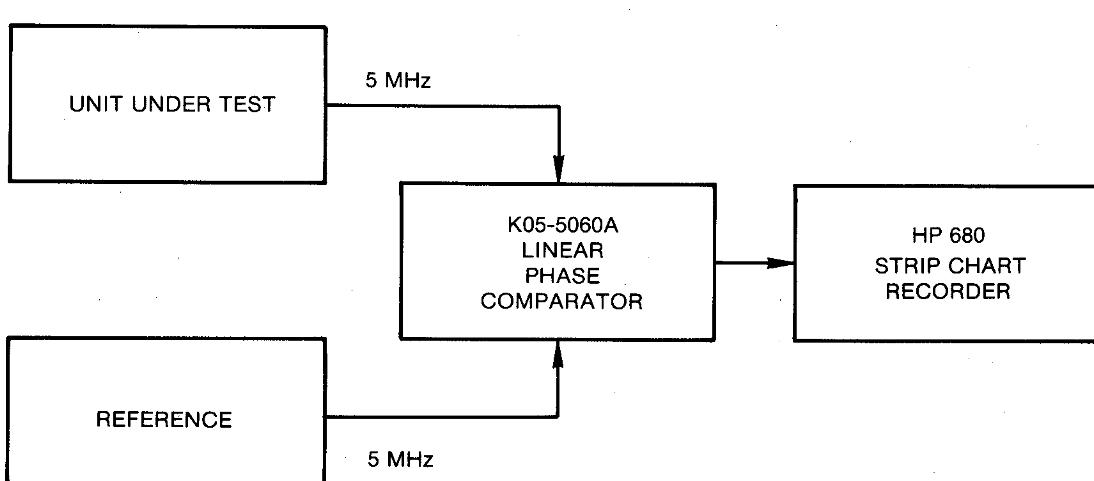


Figure 3-6. Frequency Comparison with a Phase Comparator

Because of its simplicity, understanding this method of frequency comparison merely requires an understanding of the operation of the K05-5060A. There are two input channels, one of which serves as the input for a standard signal and the other as an input for the test oscillator. A single selector switch is set at the comparison frequency; either 100 kHz, 1 MHz, or 5 MHz.

Internally, the signals on each of the two channels are compared and control the on/off duty cycle of a flip-flop circuit. The rectangular wave output from this circuit is then integrated to give an output dc signal which is proportional to the phase difference between the two input signals.

After a simple calibration of the system shown in Figure 3-6, the full scale width of the recorder chart is equal to the period of the input frequency. When comparing two 1 MHz frequencies, for example, the chart width would be 1 μs.

To compute the frequency error of the test oscillator the following relationship is used:

$$\left|\frac{\Delta f}{f}\right| = \left|\frac{\Delta t}{t}\right| = \left|\frac{\Delta \phi}{\phi}\right|$$

Which is derived in Appendix F and where,

$\frac{\Delta f}{f}$ = fractional frequency deviation of test oscillator

Δt = the measured phase difference, in microseconds

t = the elapsed measurement time, in microseconds

For example, if the recorder chart indicates a full width variation (1 μs for a 1 MHz signal) during an elapsed measurement time of 1 hour, the frequency error of the test oscillator is:

$$\left|\frac{\Delta f}{f}\right| = \left|\frac{\Delta t}{t}\right| = \frac{1 \mu s}{(60 \text{ min/hr})\ (60 \text{ s/min})\ (1 \times 10^6 \mu s/s}$$

$$= 3.2 \times 10^{-10}$$

or 3.2 parts in 10^{10} frequency error.

FREQUENCY DETERMINATION FROM TIME COMPARISONS

Before stabilized LF/VLF (Loran, Omega, etc.) transmissions were available for use in a quick and accurate calibration of a local frequency standard, it was common practice to use time ticks from a high frequency standard station such as WWV for this purpose. This time comparison method for determining frequency has fallen into disuse because it is neither quick nor convenient. In those rare cases where no access exists to LF/VLF standard signals, this method could still serve.

In the time comparison method, frequency is measured indirectly. Observations are made over an extended period of time in order to minimize errors arising from variations in propagation; overall accuracy depends on signal conditions and on the length of the test.

Suppose the local clock is driven by a precision oscillator and that its time is periodically compared with the master time by methods such as those described in Section IV. If the time intervals of the local clock precisely match those of the master clock, the oscillator frequency is precisely its nominal value. If the clock loses time, the oscillator frequency is low, if it gains time, the oscillator frequency is high.

Direct Computation. The average oscillator frequency (or average frequency error) during the elapsed time between two time comparison measurements is easily computed.

Average fractional error in frequency is equal to the fractional time error and is given by

$$\left|\frac{\Delta f}{f}\right| = \left|\frac{\Delta t}{t}\right| = \frac{t_2 - t_1}{t}$$

where $\frac{\Delta f}{f}$ = average frequency error

t_1 = initial time-comparison reading
t_2 = final time-comparison reading
t = elapsed time between readings.

Example: A time comparison reading at 10:00 a.m. on June 1 is 563,060μs; a reading at 10:00 a.m. on June 4 is 564,040μs. In this case;

$$\frac{\Delta f}{f} = \frac{564{,}040\mu s - 563{,}060\mu s}{3 \text{ days}} \times \frac{1 \text{ day}}{8.64 \times 10^{10}} = \frac{+3.8}{10^9}$$

That is, the average oscillator frequency error during this period is 3.8 parts in 10^9 high.

Average frequency of an oscillator during the measurement interval is given by

$$f_{av} = f_{nom} \left(1 + \frac{\Delta f}{f}\right)$$

where f_{av} = average frequency
f_{nom} = nominal oscillator frequency
$\frac{\Delta f}{f}$ = average frequency error

Continuing with the example given before for an oscillator with a nominal frequency of 1 MHz,

$$f_{av} = 10^6 \left(1 + \frac{3.8}{10^9}\right) = 1{,}000{,}000.0038 \text{ Hz}$$

Note that determination of oscillator frequency depends on measurement of time intervals, and does not depend on absolute time setting or time synchronization with the master time source. Because of the use of "leap seconds" in the UTC system, this technique can give erroneous results if one does not account for the leap seconds.