SECTION II
CONSIDERATIONS IN TIMEKEEPING SYSTEMS

In establishing a timekeeping system, the system designer or operator has to deal with several different sources of error. There are six major problem areas which contribute to errors in timekeeping:

1. Maintenance of accurate frequency;
2. Obtaining accurate time transfer;
3. Determination of radio propagation path delays;
4. Maximization of the frequency calibration interval;
5. Determination of the effects of noise in frequency generating equipment;
6. Determination of the effects of changing environmental conditions.

In this section, we shall discuss these problem areas in terms of their effects and methods of reducing their impact.

As mentioned earlier, it is impossible to be exact when dealing with frequency and time. However, it is possible to be 1000 times more accurate in frequency measurements than in the measurement of any other physical quantity. Therefore, prior to analyzing the effects and impact of these sources of error, it is necessary to determine the level of accuracy required and the tolerances essential for the individual application. Once the essential tolerances have been established, the sources of error can be analyzed to determine if they impact on the system operation. If they do in fact affect the system operation, then appropriate steps can be taken to reduce the impact.

ACCURATE FREQUENCY AND TIME TRANSFER

Basic to any timekeeping system is the establishment and maintenance of accurate frequency and obtaining an accurate time transfer. Inherent in the word accurate when dealing with physical measurements is the phrase: within given tolerances. Two essential ingredients are (1) stable frequency sources or clocks; and (2) a method of frequency comparison and time transfer which provides the required accuracy within given tolerances.

Section III deals with several frequency comparison techniques which can provide various levels of accuracy. Section IV identifies and compares time transfer techniques providing not only several levels of accuracy for time transfer but for frequency comparison as well.

RADIO PROPAGATION PATH DELAYS

In order to accomplish time transfer via radio waves, the radio propagation path delay has to be determined as accurately as possible. However, most techniques for initial time transfer will allow accuracies to only a millisecond or so. Normally for highly stable propagation paths (e.g. OMEGA or LORAN-C) we desire very precise time transfer within a microsecond. This level of accuracy is only possible for initial time transfer using the portable clock technique discussed in Section IV. Subsequent time transfers using highly stable propagation means can provide submicrosecond accuracies.

For those systems which do not require microsecond accuracy, Appendix A contains techniques for determining the radio propagation path delays from computation of the Great Circle Distance.

FREQUENCY CALIBRATION INTERVAL

A time system, based upon a quartz oscillator or a rubidium standard of known drift rate, can be kept within prescribed limits of error with infrequent adjustments through a systematic approach.

In this approach, the oscillator and clock are preset to offsets that will keep the time system operating within a selected accuracy for a long time despite the oscillator's drift. This drift (aging rate) must be known and must be nearly constant, so that a plot of the frequency over the adjustment interval can be approximated by a straight line. In the following, it will be assumed that the oscillator's aging rate has been established by comparisons of the oscillator against a standard.

The basic equations are presented first, then the method is illustrated with a problem solved by calculation.

Time Error vs. Frequency. The frequency at any time t can be expressed (with the frequency changes versus time approximated by a straight line):

$$f_t = f_o + af_rt \qquad \text{(Eq. 1)}$$

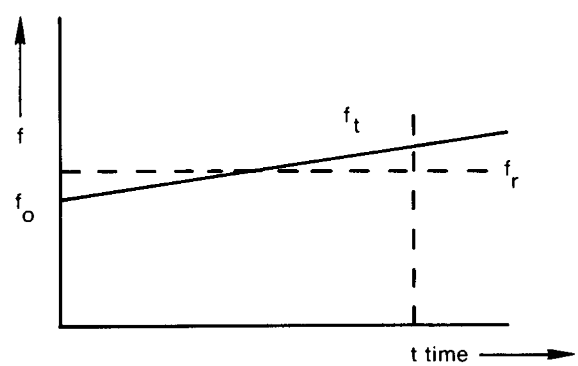

Figure 2-1. Oscillator Frequency Vs. Time

Where:

f_t = frequency at time t
f_o = initial frequency at time t=0
f_r = reference frequency (desired, zero error)
a = aging rate (fractional parts per unit time)

From the derivation contained in Appendix B the total time error is:

$$E = E_o + \left(\frac{f_o}{f_r} - 1\right) t + \frac{at^2}{2} \qquad \text{(Eq. 2)}$$

Equation 2 indicates that the total time error at any time t depends upon the values of four quantities: (1) initial time error E_O; (2) initial frequency f_O; (3) aging or drift rate a; and (4) elapsed time t.

A plot of Equation 2 as a function of time is a parabola for which vertical displacement depends upon the value of E_O, Figure 2-2. The corresponding frequency plot is shown beneath the error plot.

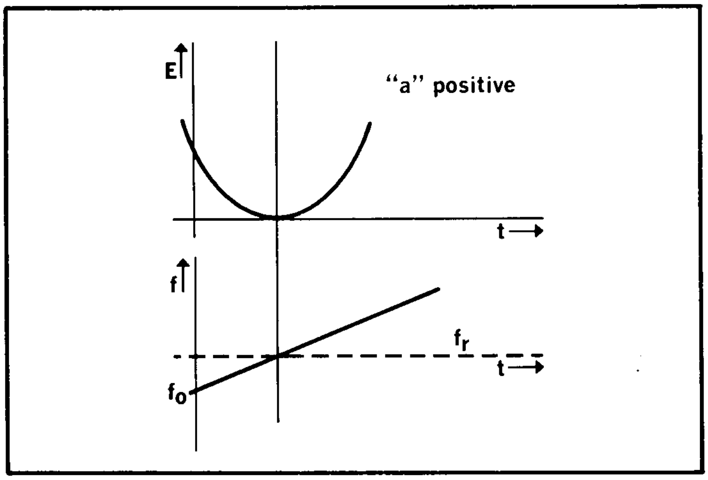

Figure 2-2. Positive Frequency Drift

Note that the oscillator frequency is precisely equal to the reference frequency at the point corresponding to the vertex of the error parabola. This is as it should be, for the slope of the curve must be zero where the two frequencies agree.

If the frequency drift were negative, the parabola would be inverted (Figure 2-3).

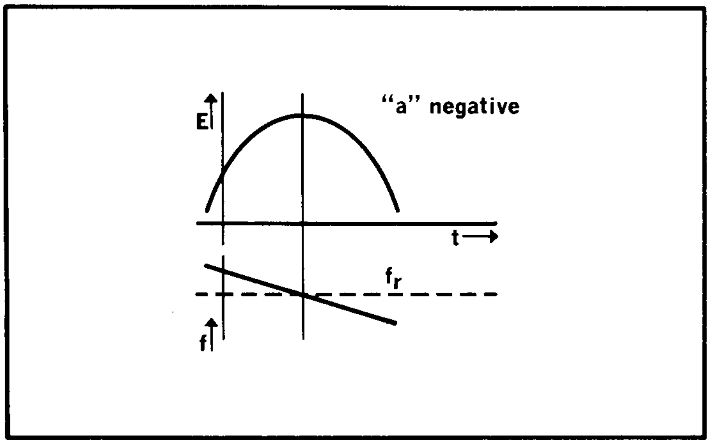

Figure 2-3. Negative Frequency Drift

Figure 2-4 shows corresponding plots of frequency and time error to clarify their relationship.

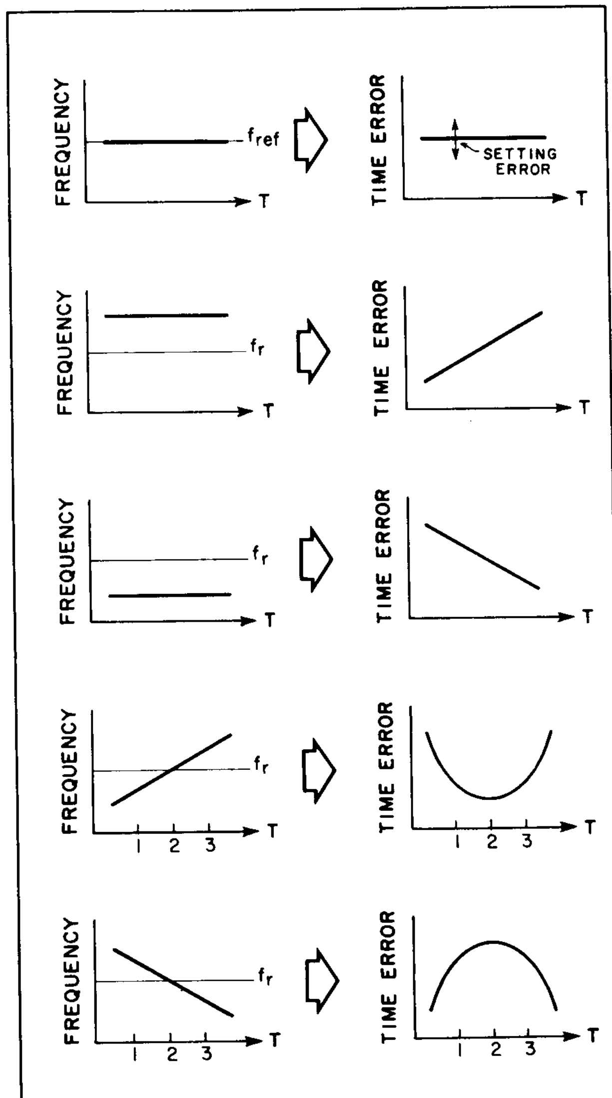

Figure 2-4. Corresponding Frequency and Time Plots

Example:
As a specific problem, consider a time system to be maintained within ±10 μs. The signal that drives the clock is derived from a rubidium standard with a known drift rate, a, of + 1 x 10^{-11}/month. The quantities to be determined are: (a) The initial time error E_0, set on the clock. (b) The initial frequency offset f_0. (c) The length of the recalibration cycle T_2, i.e., the number of days the clock is left untouched between resettings.

The elapsed time during which the error of the clock is less than 10 microseconds can be maximized by selection of initial conditions such that the error plot and the frequency plot are situated as shown in Figure 2-5.

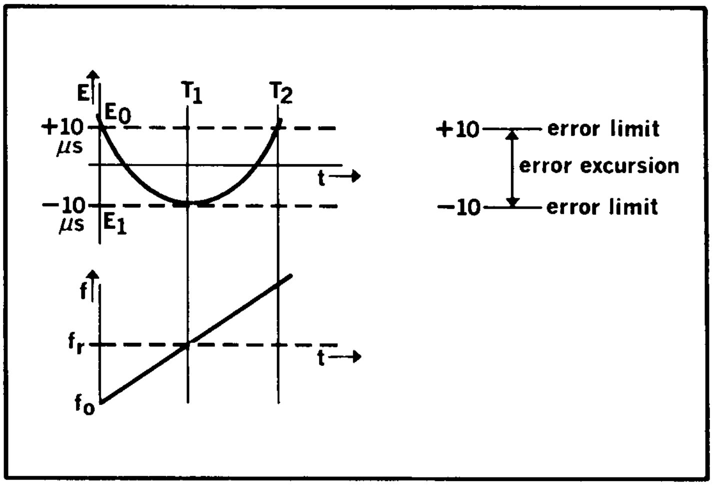

Figure 2-5. Frequency and Time Error

In the sketch, $t=T_1$ when the time error plot has a slope of zero. The parabola was positioned vertically such that its vertex at T_1 does not exceed the selected error limit, -10µs. This is accomplished by setting E_O, the initial error, at the other error limit, +10µs. We now have answer (a): $E_O = +10\mu s$.

Also, the oscillator frequency is initially set to a certain offset. These two steps maximize the elapsed time T_2 during which the system lies within the selected limits of error.

The general equation (Eq. 2) is now solved for f_O and T_2.

$$E = E_O + \left(\frac{f_O}{f_r} - 1\right) t + \frac{at^2}{2}$$

At time $t=T_1$, $E=E_1$

$$E_1 = E_O + \left(\frac{f_O}{f_r} - 1\right) T_1 + \frac{aT_1^2}{2}$$

But $E_1 = -E_O$, therefore:

$$-E_O = E_O + \left(\frac{f_O}{f_r} - 1\right) T_1 + \frac{aT_1^2}{2}$$

$$0 = 2E_O + \left(\frac{f_O}{f_r} - 1\right) T_1 + \frac{aT_1^2}{2} \qquad \text{(Eq. 3)}$$

There are two unknowns, T_1 and f_O. Since the slope is known to be zero at $t = T_1$:

$$\frac{dE}{dt} = 0 \text{ at } t = T_1$$

From Equation 1,

$$E = E_o + \left(\frac{f_o}{f_r} - 1\right) t + \frac{at^2}{2}$$

$$\frac{dE}{d_t} = 0 = \frac{f_o}{f_r} - 1 + \frac{a}{2}(2T_1) = \frac{f_o}{f_r} - 1 + aT_1$$

$$aT_1 = 1 - \frac{f_o}{f_r}$$

$$\frac{f_o}{f_r} = 1 - aT_1 \qquad \text{(Eq. 4)}$$

Substituting into Equation 3:

$$0 = 2E_o + (1 - aT_1 - 1)\,T_1 + \frac{aT_1^2}{2}$$

$$= 2E_o - \frac{aT_1^2}{2}$$

$$-4E_o = aT_1^2$$

$$T_1^2 = \frac{4E_o}{a}$$

$$T_1 = 2\sqrt{\frac{E_o}{a}} \qquad \text{(Eq. 5)}$$

The parabola is symmetric about T_1:

$$T_2 = 2T_1$$

Hence, T_2 in terms of the initial error E_o and the drift rate a, is:

$$T_2 = 4\sqrt{\frac{E_o}{a}} \qquad \text{(Eq. 6)}$$

To solve the problem numerically we substitute:

$$E_o = 10\text{x}10^{-6}\text{ sec x } \frac{1\text{ day}}{8.64\text{x}10^4\text{ sec}} = 1.16\text{x}10^{-10}\text{ day}$$

$$a = \frac{1\text{x}10^{-11}}{\text{month}} \text{x} \frac{\text{month}}{30\text{ days}} = 3.33\text{x}10^{-13}/\text{day}$$

$$T_2 = 4\sqrt{\frac{1.16\text{x}10^{-10}\text{ day}}{3.33\text{x}10^{-13}/\text{day}}} = 4\sqrt{3.48\text{x}10^2} = 75\text{ days Answer (c)}$$

The oscillator can operate for 75 days without recalibration. The oscillator's initial offset must be calculated from Equation 4:

$$\frac{f_o}{f_r} = 1 - aT_1$$

$$f_o = f_r\,(1 - aT_1)$$

But

$$T_1 = \frac{T_2}{2} = 37.5 \text{ days}$$

$$a = 1 \times 10^{-11}/\text{mo} \;\times\; 1/30 \;\text{mo/day} \;= 3.33 \times 10^{-13}/\text{day}$$

$$f_o = f_r\; 1 - (3.33 \times 10^{-13})\,(37.5) \;=\; f_r\,(1 - 1.25 \times 10^{-11}) \quad \text{Answer (b)}$$

It is clear that the oscillator must be set to a frequency lower than reference frequency by 1.25 parts in 10^{11}.

Appendix C contains recalibration charts for both quartz oscillators and rubidium standards which provide a handy reference for quick approximations of the number of days required between resettings for various levels of accuracy desired.

Regardless of whether the equation or the chart is used to determine recalibration time, it should be recognized that they are based upon perfect conditions and considerable operator skills. In reality, the environment plays a very important part in calculations of time excursions. Temperature changes, vibrations, shock, etc. can increase or decrease the frequency depending upon the individual oscillators characteristics. Also, the noise processes in the instrument, especially for the most precise tolerances, add or detract from our confidence in the answer derived. These areas are discussed later in this section.

LONG TERM EFFECTS OF NOISE

Every frequency source has a certain amount of noise inherent in the circuitry and components. The noise generated causes effects in the short-term stability and the long-term stability. The effects of noise on the short-term stability will be covered in AN 52-3, Stability: Theory and Measurement. The long term effects of noise are important in timekeeping applications.

We assume a frequency calibration (measurement) time interval, T_c, during which the system clock is compared against a reference to determine its average frequency. Following a dead time, T_d, the phase or time of the clock is measured and we wish to estimate the variance in the indicated time after an interval, T_p, where we assume the correction determined during the calibration is applied.

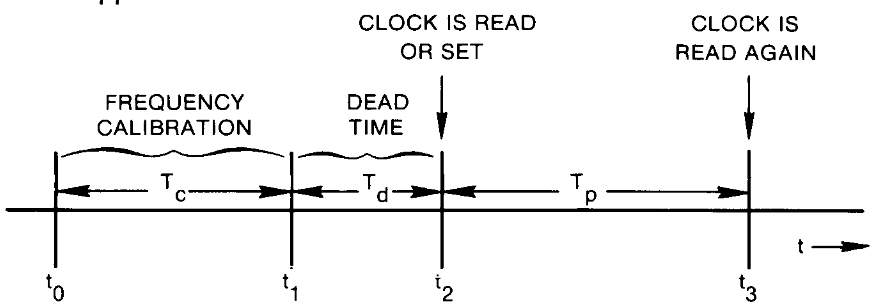

Figure 2-6. Variance in Time Interval

Appendix D, Variance of Time Interval for Calibrated Clocks derives a set of equations which can be used to estimate the variance. There are two types of noise in the instrument which affect the accumulation of time and produce uncertainties in the predictability of a given set of clocks. The two types of noise are white FM and flicker FM.

For an example, let us examine two HP 5061A, Option 004 High Performance Cesium Standards which are intercompared for 10 days and then the clocks are intercompared again after 60 days. What is the one-sigma certainty that we predict the time difference in sixty days?

White FM component: from equation 5 in Appendix D.

$$\overline{E_W^2} = \frac{A}{2}\left(T_p + \frac{T_p^2}{T_c}\right)$$

where

T_c = 10 days = 8.64×10^5 sec
T_p = 60 days = 5.184×10^6 sec

and for Option 004:

$$A = 1.28 \times 10^{-22} \text{ sec}$$

$$\overline{E_W^2} = \frac{1}{2}\,(1.28 \times 10^{-22})\left[5.184 \times 10^6 + \frac{(5.184 \times 10^6)^2}{8.64 \times 10^5}\right]$$

$$\overline{E_W^2} = 2.32\times10^{-15} \text{ sec}^2$$

Flicker FM component: from equation 6, Appendix D.

$$\overline{E_f^2} = BT_p^2\left\{\frac{(T_p + T_d + T_c)^2}{T_pT_c}\ln\left(1 + \frac{T_p + T_d}{T_c}\right)\right.$$

$$+\frac{T_d^2}{T_cT_p}\ \ln\frac{T_d}{T_c} - \ln\frac{T_p}{T_c}$$

$$-\frac{(T_p + T_d)^2}{T_p\,T_c}\ \ln\ \frac{(T_p + T_d)}{T_c}$$

$$\left.-\frac{(T_c + T_d)^2}{T_p\,T_c}\ln\left(1 + \frac{T_d}{T_c}\right)\right\}$$

where $T_d = 0$

and for Option 004: $B = 6.5 \times 10^{-28}$

$$\overline{E_f^2} = (6.5\text{x}10^{-28})\ (5.184\text{x}10^{6}\ \text{sec})^2 \left\{ \frac{(60+0+10)^2}{(60)\ (10)} \ell n \left(1 + \frac{60+0}{10}\right) \right.$$

$$+ \frac{0}{(60)\ (10)}\ \ell n\ \frac{0}{10} - \ell n \frac{60}{10}$$

$$\left. - \frac{(60+10)^2}{(60)(10)}\ \ell n \frac{(60+0)}{10} - \frac{(60+0)^2}{(60)(10)}\ \ell n\ (1 + \frac{0}{10}) \right\}$$

NOTE: We can use days instead of seconds inside the brackets because the seconds conversion will cancel.

$$E_f^2 = (1.7468\text{x}10^{-14}\ \text{sec}^2) \left\{ \frac{70^2}{600}\ \ell n\ 7 + 0 - \ell n\ 6 \quad - \frac{70^2}{600}\ \ell n\ 6 - 0 \right\}$$

$$= 5.85 \times 10^{-14}\ \text{sec}^2$$

The total variance is computed from the two components by:

$$\overline{E_T^2} = \overline{E_w^2} + \overline{E_f^2}$$

$$= 2.32 \times 10^{-15}\ \text{sec}^2 + 5.85 \times 10^{-14}\ \text{sec}^2$$

$$= 6.08 \times 10^{-14}\ \text{sec}^2$$

$$E_T = 2.466 \times 10^{-7}\ \text{sec}$$

$$E_T = 246.6\ \text{nsec}$$

Therefore, the one-sigma certainty of predicting the time difference 60 days from the start point is ±246.6 nsec.

For the second example let us examine a timekeeping system where the user wants to determine the maximum that four high performance cesium standards (HP 5061A, Option 004) can remain within 1μsec of each other.

To simplify the problem, we designate one of the clocks as the master and compare the other three to it. Secondly, we assume that one of the three clocks will be worse than the others and therefore we can reduce the basic problem to one involving two clocks. Using the 10638A Degausser with the High Performance Cesium Standards allows a settability of 1×10^{-13}. Once the clocks are set, at the end of the comparison time, T_C, we would expect zero frequency shift for CONSTANT environment. However, we know noise affects the overall time accumulation. Figure 2-7 shows the problem in a graphical form. We find the error components for settability and noise and add to find the total error.

First, let us examine the settability error. In an ideal case, the variance, E_{T1} is zero. Therefore the maximum time to remain within 1μsec is simply related to the settability (with CONSTANT environment) from (derived in Appendix F)

$$\left|\frac{\Delta t}{T}\right| = \left|\frac{\Delta f}{f}\right|$$

we find

$$T_{ideal} = \frac{\Delta t}{\frac{\Delta f}{f}} = \frac{1\mu sec}{1x10^{-13}} = 10^7 \text{ sec} = 115.7 \text{ days}$$

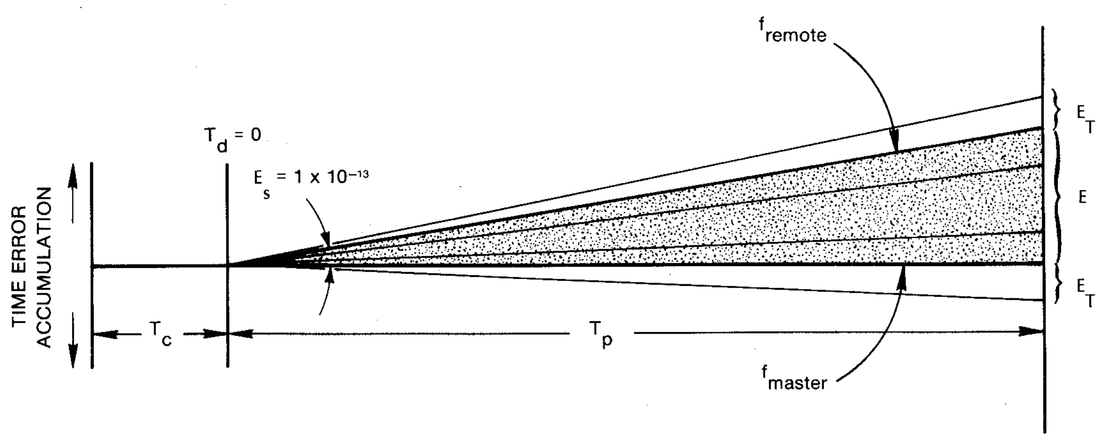

Figure 2-7. Comparison of Two Clocks

Second, we need to determine the noise component of the error. To compute the one sigma probability of remaining within 1μsec (again assuming CONSTANT environment) we have to satisfy the following equation.

$$E + 2E_T = 1\mu sec \qquad \text{(Eq. 7)}$$

where

$$E = E_s T_p$$

$$E_s = \frac{\Delta f}{f}$$

and

$$E_T = \sqrt{\overline{E_w^2} + \overline{E^2_f}}$$

The factor of 2 in Equation 7 assumes the unlikely worst case, that the two clocks' random errors are identical and opposite, i.e. the noise processes have a correlation coefficient of –1. If the two clocks are uncorrelated (likely) the factor $\sqrt{2}$ should be used.

A programmable calculator will greatly assist in the determination of T_p (max) through repeated iterations. If we assume a T_c of 10 days and $T_d = 0$, the iterations yield T_p = 60 days. With T_p = 60 days; E_T = 246.6 nsec as shown in the previous example.

$$E_f = E_s T_p + 2E_T$$

$$= 518.4 \text{ nsec} + 2(246.6) \text{ nsec}$$

$$= 1.011 \mu\text{sec}$$

which is barely over the 1μsec allowable.

EFFECTS OF ENVIRONMENTAL CONDITIONS

Environmental conditions are usually a major cause of error in precision timekeeping. For ultra-precise timekeeping, a controlled environment is the best. For real-time, transportable systems, the environment can be modeled and compared against environmental specifications to determine short and long-term effects on timekeeping. The prediction of timekeeping ability (in terms of tolerances allowable) is then directly related to the accuracy of the environmental model and the actual performance of the individual frequency sources.

The most conservative system design approach is to assume the worst case specified environmental sensitivities for the instrument and compare to the system specifications. An alternate scheme requires constant monitoring of the timekeeping systems performance to insure that environmental changes will be compensated or corrected.

For ultimate results in timekeeping, a computer modeling may be required to accurately predict the suitability of a given frequency source to a given set of environmental conditions. The environmental variables needed in the modeling include:

1. Effects of temperature and temperature changes on the frequency;
2. Effects of altitude and altitude changes on the frequency;
3. Effects of magnetic fields (AC and DC) on the frequency;
4. Effects of humidity on the frequency;
5. Effects of shock and vibration on the frequency;
6. Effects of gravity and orientation on the frequency;
7. Interdependency of effects.

Not only do these conditions affect the frequency of an oscillator, they can also affect the phase (e.g., the phase shift in an amplifier might change with temperature). This complicates the measurement of model parameters and affects the resultant model.