Forward Error Correction and Pictures From Mars

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September 2020

Introduction

- Forward Error Correction (FEC) codes
- Hadamard linear block code
  - No complex math explanations
- Simple example
  - One-way broadcast of secret message to agent
- Mariner 9 Spacecraft example
  - Achieved Martian obit Nov 1971 — First Earth-made object to orbit another planet
  - Transmitted pictures of the Martian surface until Oct 1972
  - Picture data required Forward Error Correction

Forward Error Correction (FEC)

Reduces errors in poor communication channel

- FEC used:
  - When no reverse channel available to request retransmission
  - One way broadcasts
- FEC is achieved by redundant encoding of data
  - Errors can be detected and corrected anywhere in the encoded data
  - Maximum errors corrected is determined by design of FEC
- FEC cost is increased time or bandwidth to send data

FEC Types

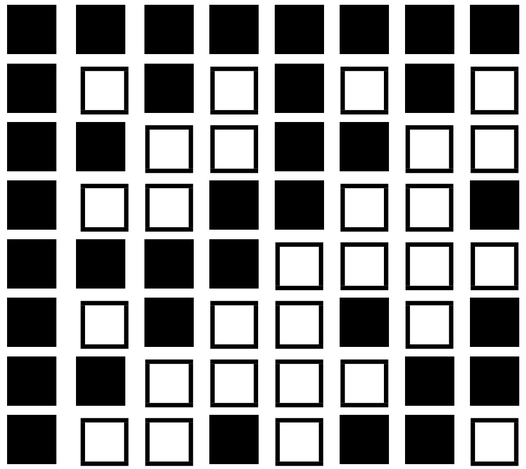
- Linear Block Codes
  - Encode Data Words
- Convolutional Codes
  - Encodes continuous steam of data
- This presentation limited to Hadamard linear block code FEC
  - Also called Walsh code, Walsh-Hadamard code
  - Code Words are the rows of a Hadamard matrix

Hadamard Matrix

- Studied for 1½ centuries
- New uses still being discovered
- Walsh code for CDMA

$$\left[\begin{array}{cccc|cccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
\hline
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1
\end{array}\right]$$

1 1 1 1 1 1 1 1
1 0 1 0 1 0 1 0
1 1 0 0 1 1 0 0
1 0 0 1 1 0 0 1
1 1 1 1 0 0 0 0
1 0 1 0 0 1 0 1
1 1 0 0 0 0 1 1
1 0 0 1 0 1 1 0



0 0 0 0 0 0 0 0
0 1 0 1 0 1 0 1
0 0 1 1 0 0 1 1
0 1 1 0 0 1 1 0
0 0 0 0 1 1 1 1
0 1 0 1 1 0 1 0
0 0 1 1 1 1 0 0
0 1 1 0 1 0 0 1

Simple Example, Transmit

- Broadcast station transmit Data Words in the range of 000 to 111
- Covert receive station needs 4 Data Words to identify mission
  - This example, covert agent mission is number 3333
- Broadcast station needs to encode and transmit 4 times the Data Word 011

Encode Table

Data Value	Hadamard matrix	Data Word
0	11111111	000
1	10101010	001
2	11001100	010
3	10011001	011
4	11110000	100
5	10100101	101
6	11000011	110
7	10010110	111

Simple Example, Receive

Column Headings Definitions for Decode Table

Send Data Value = 3 and Data Word = 011

Transmitted Data Code Word = 10011001

RCWx = Received Code Word with x errors

RCW0 = 10011001 (0 error.)

RCW1 = 10001001 (1 error. red indicates received bit error)

RCW2 = 10000001 (2 errors. red indicates received bit error)

RCW3 = 10000101 (3 errors. red indicates received bit error)

Simple Example, Receive

- Received Code Word score
  - 1 point for same bit value in the same position as a row in the matrix
- Correct a maximum of 1 error
- Detect a maximum of 2 errors
- Incorrect decode for 2 or more errors

Decode Table					
Data Value	Hadamard Matrix	RCW0 score	RCW1 score	RCW2 score	RCW3 score
0	11111111	4	3	2	3
1	10101010	4	5	4	3
2	11001100	4	5	4	5
3	10011001	8	7	6	5
4	11110000	4	3	4	3
5	10100101	4	5	6	7
6	11000011	4	5	6	5
7	10010110	4	3	4	5

Mariner 9 Background

- Pictures of Mars surface
  - Black and white
  - Each photo about 4 to 5 km square
  - 4.5 million bits/picture
    - Split into 6 bits Data Words
    - 64 different Data Word

Mariner 9 FEC Option

- Maximum length for FEC Code Word was about 30 bits
- 5 repeat code was a possibility
  - 6 bit Data Word encoded with 5 repeat code = 30 bits
    - Example 101010 becomes 111111000001111100000111111000000
  - Advantage = easy implementation
  - Disadvantage = Correct 2 bit error

Mariner 9

Selected Hadamard Code

- Correct 7 bit errors per Code Word
- No Spacecraft memory or processor required to generate Code Word
  - Using logic circuits, Code Word generated as Data Word was available
- Rapid decoding using a black box called “The Green Machine”
- Used two 32 X 32 Hadamard matrices for 6 bit Data Words
  - Augmented Hadamard code (64 X 32)
    - Data Word with MSB = 1, Code Word from top half 64 X 32
    - Data Word with MSB = 0, Code Word from bottom half 64 X 32

Mariner 9

Data & Code Words

- All Data Words in the Mariner 9 Dictionary
- Data Words not in order

Data Word	Code Word
111111	11111111111111111111111111111111
100000	10101010101010101010101010101010
110000	11001100110011001100110011001100
101111	10011001100110011001100110011001
111000	11110000111100001111000011110000
100111	10100101101001011010010110100101
110111	11000011110000111100001111000011
101000	10010110100101101001011010010110
111100	11111111000000001111111100000000
100011	10101010010101011010101001010101
110011	11001100001100111100110000110011
101100	10011001011001101001100101100110
111011	11110000000011111111000000001111
100100	10100101010110101010010101011010
110100	11000011001111001100001100111100
101011	10010110011010011001011001101001
111110	11111111111111110000000000000000
100001	10101010101010100101010101010101
110001	11001100110011000011001100110011
101110	10011001100110010110011001100110
111001	11110000111100000000111100001111
100110	10100101101001010101101001011010
110110	11000011110000110011110000111100
101001	10010110100101100110100101101001
111101	11111111000000000000000011111111
100010	10101010010101010101010110101010
110010	11001100001100110011001111001100
101101	10011001011001100110011010011001
111010	11110000000011110000111111110000
100101	10100101010110100101101010100101
110101	11000011001111000011110011000011
101010	10010110011010010110100110010110

Data Word	Code Word
010101	01101001100101101001011001101001
001010	00111100110000111100001100111100
011010	01011010101001011010010101011010
000101	00001111111110000111100000001111
010010	01100110100110011001100101100110
001101	00110011110011001100110000110011
011101	01010101101010101010101001010101
000010	00000000111111111111111100000000
010110	01101001011010011001011010010110
001001	00111100001111001100001111000011
011001	01011010010110101010010110100101
000110	00001111000011111111000011110000
010001	01100110011001101001100110011001
001110	00110011001100111100110011001100
011110	01010101010101011010101010101010
000001	00000000000000000111111111111111
010100	01101001100101100110100110010110
001011	00111100110000110011110011000011
011011	01011010100101010110101010100101
000100	00001111111100000000111111110000
010011	01100110100110010110011010011001
001100	00110011110011000011001111001100
011110	01010101101010100101010110101010
000011	00000000111111110000000011111111
010111	01101001011010010110100101101001
001000	00111100001111000011110000111100
011000	01011010010110100101101001011010
000111	00001111000011110000111100001111
010000	01100110011001100110011001100110
001111	00110011001100110011001100110011
011111	01010101010101010101010101010101
000000	00000000000000000000000000000000

Mariner 9

Numeric example

- Blue for Data Words with 1=MSB
- Green for Data Words with 0=MSB
- Red for received bit errors
- First Received Code Word
  101010100101010110101010010101010
- Last Received Code Word
  000000001111111100000000000000000

Send Data Words alternates between 100011 and 000011

Transmitted Code Words alternates between:

101010100101010110101010010101010
000000001111111100000000011111111

First Received Code Word is 1E0 and last is 0E8
1Ex = x errors for Received Code Word that has 1 as MSB
0Ex = x errors for Received Code Word that has 0 as MSB

Received Code Words. Red bit values are bit errors.

1E0 = 101010100101010110101010010101010
0E0 = 000000001111111100000000011111111
1E1 = 101010100101010110101010010101000
0E2 = 000000001111111100000000011111100
1E3 = 101010100101010110101010010100010
0E4 = 000000001111111100000000011110000
1E5 = 101010100101010110101010010001010
0E6 = 000000001111111100000000011000000
1E7 = 101010100101010110101010001101010
0E8 = 000000001111111100000000000000000

Note: The position of any bit error does not affect the score

Mariner 9 - Partial Decode Table

Data Word	Code Word	1 E0	0 E0	1 E1	0 E2	1 E3	0 E4	1 E5	0 E6	1 E7	0 E8
101000	10010110100101101001011010010110	16	16	17	16	17	16	15	16	15	16
111100	11111111000000001111111100000000	16	0	17	2	17	4	17	6	17	8
100011	10101010010101011010101001010101	32	16	31	16	29	16	27	16	25	16
110011	11001100001100111100110000110011	16	16	15	14	17	16	15	14	17	16
101100	10011001011001101001100101100110	16	16	17	16	17	16	17	16	17	16
001100	00110011110011000011001111001100	16	16	17	18	15	16	17	18	15	16
011110	01010101101010100101010110101010	0	16	1	16	3	16	5	16	7	16
000011	00000000111111110000000011111111	16	32	15	30	15	28	15	26	15	24
010111	01101001011010010110100101101001	16	16	15	16	15	16	17	16	17	16
001000	00111100001111000011110000111100	16	16	17	18	15	16	15	14	17	16

Hadamard Code Bit Error Rate (BER)

- No special test equipment required for BER measurements
- BER = $\dfrac{\text{(Total number of corrections for all transmitted Code Words)}}{\text{(Total number of transmitted bits)}}$
- BER for Mariner 9 example
  Average BER = (0+0+1+2+3+4+5+6+7+8)/(10*32) = 36/320 = .11 = 11%
- Total number of corrections in a code word must be less than the total number of maximum detectable errors for a code word.
  - For Mariner 9, maximum detectable errors = 8

Summary

- Hadamard code improves data transmission over a bad communication link
- HF digital mode Olivia:
  - Uses Hadamard FEC
  - Two way chat communications (like RTTY)
  - Decodes well
    - -14 db SNR
    - Works through atmospheric noise, fade, interference

Questions?