Baseband Group Delay Equalization of IF Filters for Data Communications

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Abstract

One source of phase (and thus group delay) variation is the IF filter in the receiver. This is either a mechanical filter or more commonly, a crystal filter. These IF filters typically have been optimized for best amplitude rejection of out-of-band signals vs. achievable passband flatness for a given number of filter poles. This typically leads to a Chebychev filter implementation which can have significant group delay variation. If the group delay variation vs. frequency is significant compared to the symbol rate (in baud) then the bit-error-rate of the received signal may be substantially degraded. Techniques to equalize the filter at IF are possible, but may prove difficult to implement and adjust, however they work well independent of the modulation type. On the other hand equalization of the filter delay at baseband is possible, but is only exact for linear modulation types (PSK, QAM). FSK modulation (which is non-linear) cannot be exactly compensated at baseband but if the deviation index is low, then a reasonably good job of delay equalization at baseband can be accomplished. This article examines some amplitude, phase, and delay properties of first-order, second-order and all-pass filters, and illustrates examples of Chebychev and Butterworth IF filters. Finally, a graphical representation of delay equalization at baseband is shown.

Introduction

It can be shown that there exists several optimal channel amplitude responses for data communications channels, one of the more popular being the raised-cosine channel response. It can also be shown that the optimum bit-error-rate (BER) vs. received power level occurs when the channel phase is linear (group delay is flat)[1]. The optimum amplitude response vs. frequency has been discussed in a previous paper[2], this paper will examine the flat group delay requirement, and equalization of IF filter group delay variation.

Filter Transfer functions

All linear filtering functions can be described as a transfer function relating the filter's output amplitude and phase with respect to the filter's input amplitude and phase. For circuits using reactive components (inductors, capacitors), the transfer function depends on frequency, and includes terms involving complex numbers. The transfer function is usually abbrevi'ated H(s), where H is a function of the variable, s. In performing analysis of the filter to a sine wave input, we substitute for s:

$$s = j\omega = j2\pi f \tag{1}$$

where j is equal to the square-root of negative 1. That is, j is the imaginary operator, f is the frequency in hertz, and omega is the frequency in radians/second. In order to describe the properties of a filter, it is necessary to derive the transfer function and then analyze the transfer vs. frequency. To describe the transfer function of a network, it is necessary to describe the complex impedance of each of the components, and then describe the output to input transfer in those terms, The complex impedance of an inductor is given by:

$$Z(s) = j2\pi fL = sL \tag{2}$$

This shows that the impedance of an inductor increases with frequency, and that there is a +90 degree phase shift associated with the inductor (the +j term). Similarly, the impedance of a capacitor is given by:

$$Z(s) = \frac{1}{j2\pi fC} = \frac{1}{sC} \tag{3}$$

which shows that the impedance of a capacitor decreases with frequency, and there is a -90 degree phase shift associated with the capacitor (the 1/j term, since l/j = -j). The impedance of a resistor is just R of course, independent of frequency and with no associated phase shift.

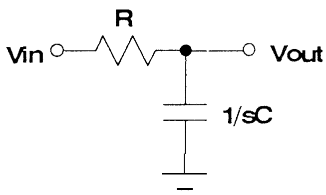


Figure 1 - First-order. low-pass filter

To derive the transfer function of a first-order low-pass filter (figure 1), we can write the voltage divider equation :

$$H(s) = \frac{\frac{1}{SC}}{R + \frac{1}{SC}} = \frac{1}{RsC + 1} \tag{4}$$

If we assume that R= 1 ohm, and C=1 farad, then the time constant is one second, and RC = 1. The we can *normalize* the response to 1 second, which is a filter -3 dB. cutoff of 1 radian/second. This can be expressed as:

$$H(s) = \frac{1}{s+1} \tag{5}$$

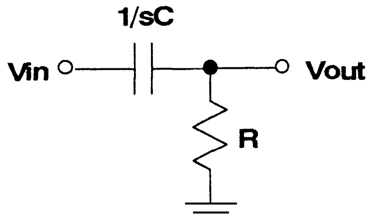


Figure 2 - First-order high-pass filter

Similarly, we can derive the transfer function for a first-order high-pass filter response (figure 2):

$$H(s) = \frac{R}{R + \frac{1}{sC}} = \frac{RsC}{RsC + 1} \tag{6}$$

When normalized to R=1 ohm, and C= 1 farad, this is simplified to:

$$H(s) = \frac{s}{s+1} \tag{7}$$

An interesting active circuit is the all-pass filter (figure 3). We can derive the transfer function for this circuit in a manner similar to above, and when normalized to R=1 ohm and C=1 farad, the transfer becomes:

$$H(s) = \frac{s-1}{s+1} \tag{8}$$

It can be seen that the amplitude does not change with frequency, s, but the phase response does. This circuit can thus be used to compensate phase delay variation (and thus group delay variation) without affecting the amplitude response.

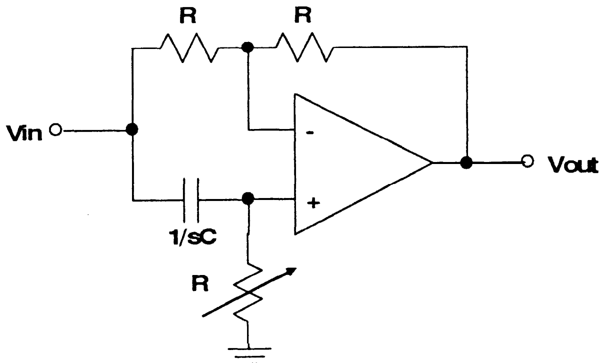


Figure 3 - First-order all-pass filter

The group delay of a transfer function is related to the phase slope, steeper phase change means greater group delay. Assuming phi is the phase in radians and omega is the frequency in radians/second, then group delay, in seconds, is:

$$group\ delay = -\frac{d\varphi}{d\omega} \tag{9}$$

If the units are degrees and hertz, then the group delay in seconds is (degrees per hertz)/360. If the phase is linear (changes at the same rate regardless of frequency) then the group delay is constant vs. frequency. To analyze and plot these three transfer functions, an Excel™ spreadsheet was set up to evaluate the amplitude, phase, and group delay of each. Since Excel can directly manipulate complex numbers, this is relatively easy to do.

Figure 4 illustrates the amplitude and phase response of the low-pass filter, figure 5 illustrates the amplitude and phase of the high-pass filter. Figure 6 illustrates the phase and group delay of the low-pass and high pass filters. The two filters have the same phase shape, even though there is a difference in the absolute phase. This absolute phase difference has no effect on the group delay, which is identical for both filters.

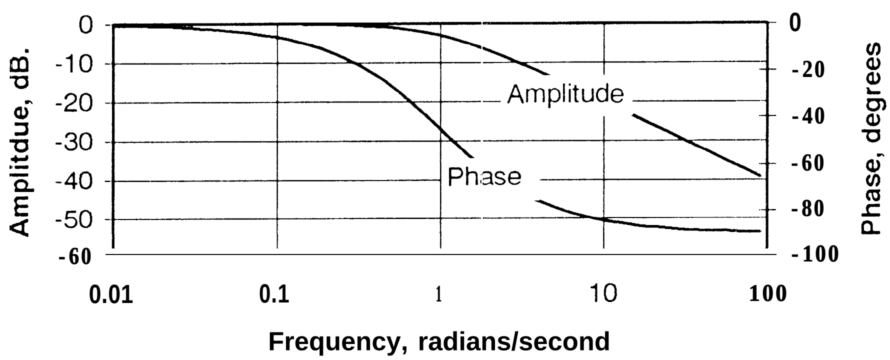


Figure *4* - Amplitude and phase of a low-pass filter

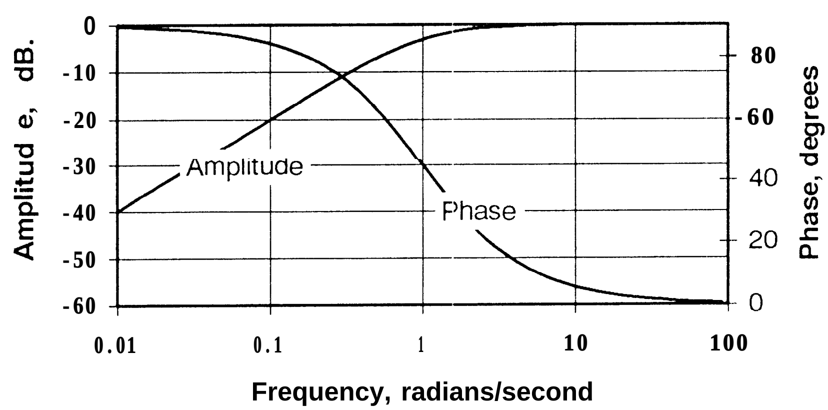


Figure 5 - Amplitude and phase of a high-pass filter

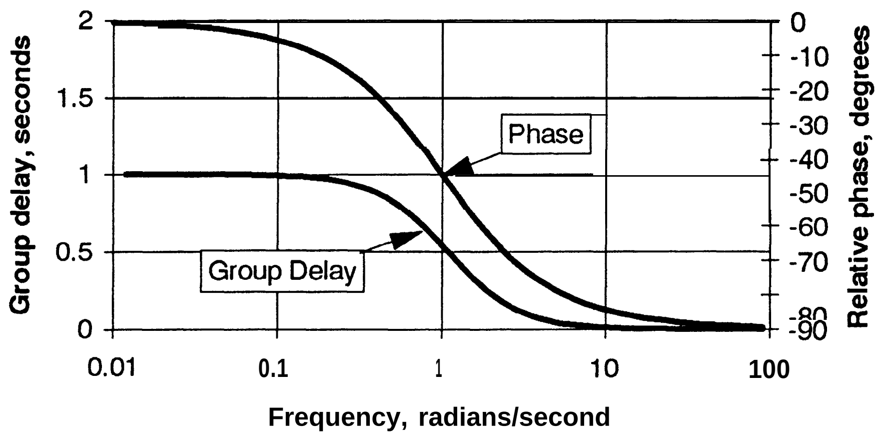


Figure 6 - Phase and Group delay of low-pass and high-pass filters (they have exactly the same **group** delay)

The all-pass filter has flat amplitude response, which is not shown. The phase response and group delay are shown in figure 7.

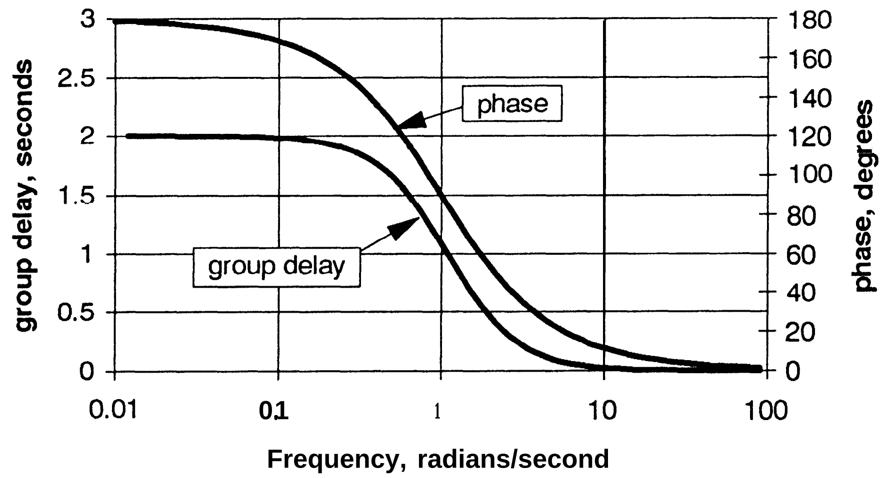


Figure 7 - Phase response and group delay of the all-pass filter, with a break frequency of 1 radian/second.

As the frequency response of the all-pass is adjusted, the phase and delay responses change. Figure 8 shows the phase and delay response of a the all-pass filter when the variable resistor of figure 3 is adjusted for a break frequency of 3 radians/second. Note that the absolute delay at low frequency is only one-third as large as in figure 7.

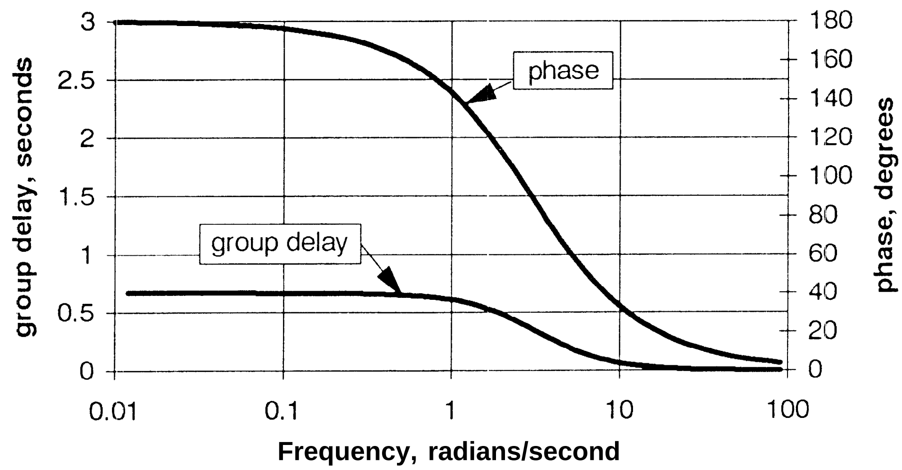


Figure 8 - Phase and Group Delay response of all-pass filter, with a break frequency of 3 radians/second

Higher Order Filters

Higher order filters, such as band-pass filters can be analyzed just like the low-, high-, and all-pass filters. A simple bandpass filter composed of a series LC and a shunt R yields the amplitude, phase, and group delay response as shown in figure 9. This filter has a resonant frequency of 1 radian/second, and a damping of 0.707 (a Q-factor of 1.414). Q is the inverse of damping.

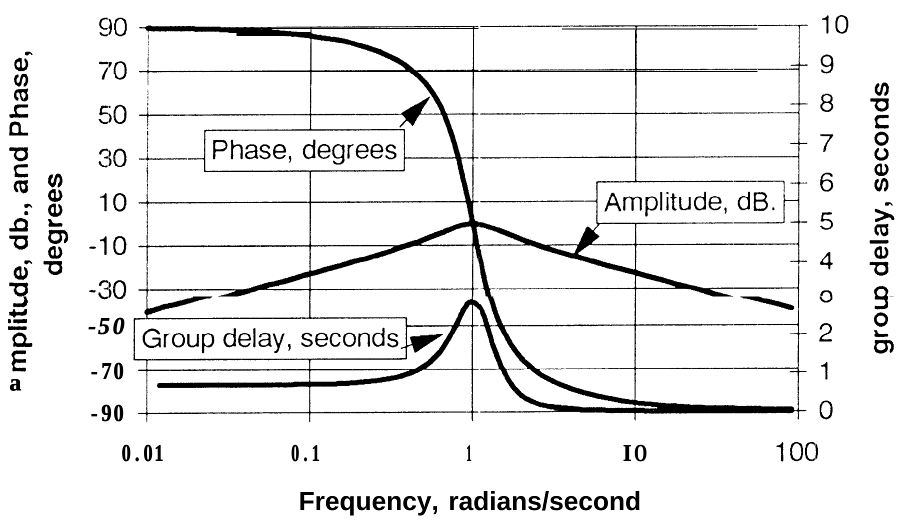


Figure 9 - Amplitude, Phase, and Group Delay response of a bandpass filter

Here it can be seen that the group delay has a peak at the center frequency of the filter. As the damping of the filter is changed, the group delay also changes, with low damping (high-Q) filters having more group delay at the peak. When constructing more complex bandpass filters, the delay will peak at the corners of the filter. The sharper the corners, the greater will be the group delay, generally. To illustrate with two examples, the frequency and group delay response of a 6-pole Butterworth, a 6-pole Chebychev, and an 8-pole Chebychev filter will be analyzed. These filters have been moved up to around 12 radians/second center frequency.

Analysis of Butterworth and Chebychev Bandpass Filters

The transfer function, H(s) for these filters can be constructed and analyzed just as has been done previously. The transfer function of the 6-pole filters is described by equation 10:

$$H(s) = \frac{-(s-z_1)(s-z_2)(s-z_3)}{(s-p_1)(s-p_2)(s-p_3)} \tag{10}$$

Where p1, p2, p3 are the ***poles*** of the filter response, and zl, z2, z3 are the zeros of the filter response. The zeros determine where the filter response goes to zero (no output), and the poles determine where the filter response peaks (maximum output). For the Butter-worth and Chebychev filters, all three zeros are at zero hertz, so equation 10 can be simplified to equation 11:

$$H(s) = \frac{-s^3}{(s-p_1)(s-p_2)(s-p_3)} \tag{11}$$

The Butterworth filter has a passband response that is maximally-flat, and the Chebychev Type-I filter has an equiripple passband response (the Chebychev Type-II has an equiripple stopband response). The difference between these two filter types is the placement of the poles. In a Butterworth filter, the poles are placed in a circle on the s-plane, and in a Chebychev filter, they are placed on an ellipse. The two axis of the s-plane correspond to the real and the imaginary parts of the pole. Since a pole position may in fact be complex, then the filter pole is actually a complex pole-pair (one pole at positive frequency, the other pole at negative frequency). The filter described by equation 11 is a 3-order filter, which has 3 complex pole-pairs, and is referred to as a 6-pole filter. Thus the 6-pole Butterworth bandpass filter consists on two sets of 3 pole pairs on two circles, one with it's center at the filter center frequency, the other with it's center at the negative of the filter center frequency. Each of the 3 poles lies on the half of the circle that lies in the left-hand plane. Figure 10 is a pole-zero diagram of the Butterworth filter. The 3 zeros are piled up on top of each other at the origin.

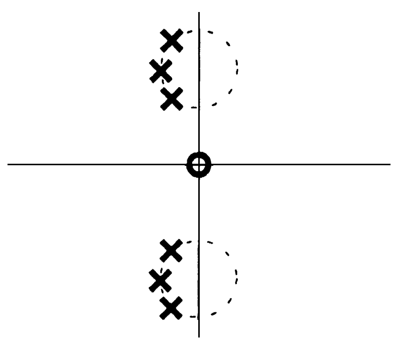


Figure 10 - pole / zero diagram of the 6-pole Butterworth filter.

Equation **10** is input to an Excel spreadsheet, and the amplitude and group delay plotted as a function of frequency. Figure 11 is the 6-pole Butter-worth response, figure **12** is the 6-pole Chebychev response, and Figure **13** is the 8-pole Chebychev response.

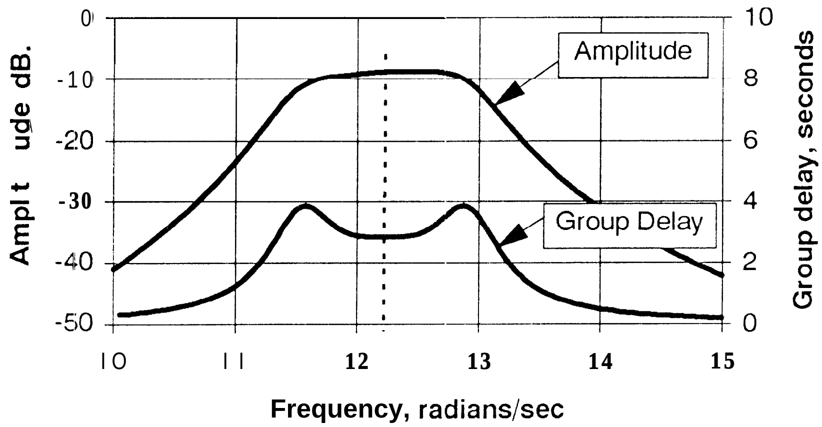


Figure 11 - 6-pole Butterworth amplitude and group delay response,

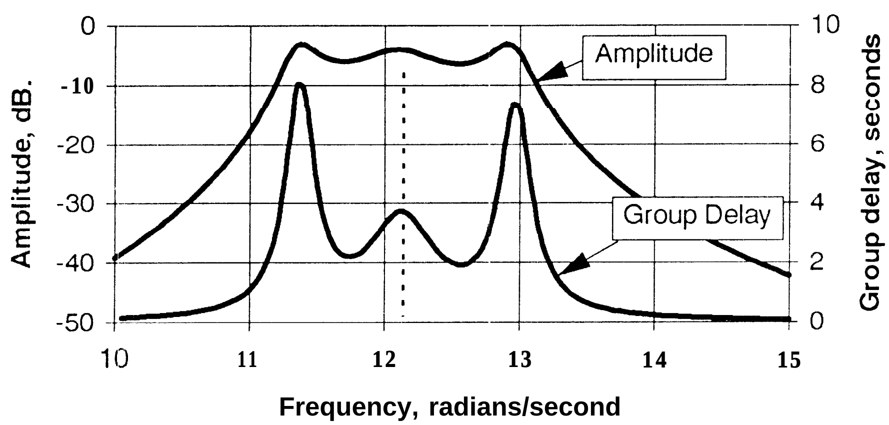


Figure 12 - 6-pole Chebychev amplitude and group delay response.

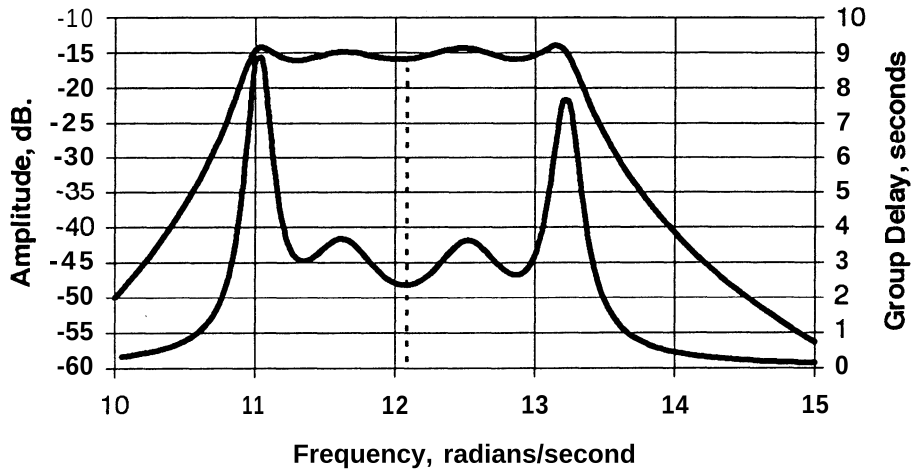


Figure 13 - 8-pole Chebychev amplitude and group delay response.

Delay equalization

It can be seen from the three filters that the group delay is largest at the 'corner' of the filter. The vertical dashed line represents the center frequency of the filter. Since each of these filters is at the Intermediate Frequency, or IF of the radio, the center frequency of the filter will be demodulated down to zero hertz at baseband by the last mixer (or by the discriminator in an FM detector). Frequencies to the left of the center frequency are negative frequencies at baseband, those to the right are positive frequencies. At baseband, it is impossible to distinguish the positive from the negative frequencies, and they all look like positive frequencies. Thus the IF response, when *translated to* baseband appears as that part of the response that lies at the filter center frequency and towards the right of that point.

Thus the task of group delay equalization is to impart significant delay at low frequencies, and less delay at higher frequencies, hopefully the inverse of the filter's baseband delay characteristic. Referring back to figures 7 and 8, it can be seen that the all-pass filter has exactly this needed delay characteristic. Thus using one of these all-pass filters at baseband in the receiver should allow us to cancel some of the filter delay. It may require several cascaded all-pass filters to achieve the necessary delay, and some of the all-pass filters may have to be adjusted to different break frequencies in order to properly compensate for the IF filter's baseband delay characteristic.

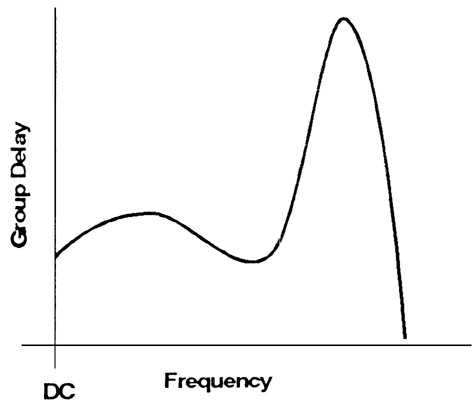
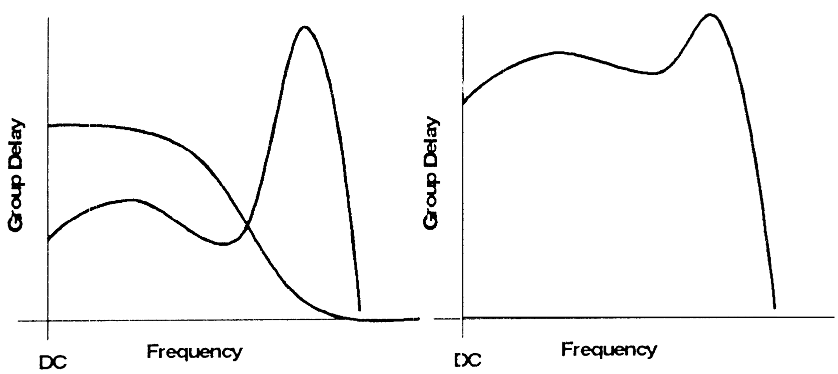


Figure 14 - Baseband delay of Chebychev filter

Figure 15a shows the delay of the all-pass filter chain and the Chebychev filter on the same graph, and figure 15b shows the total delay of the all-pass and Chebychev in series.



A B

Figure 15 - A) delay of all-pass filter and Chebychev filter on the same graph
B) total (sum) of the delay for the equalizer and filter.

It can be seen from figure 15b that the total delay variation *in the region* of *interest* is less than in figure **14.** The region of interest includes the passband of the filter, but not the stopband of the filter. Since the received signal that is outside the passband is rejected by the filter, it's group delay is not important.

Required Filter Bandwidth

The required IF filter width, when properly group delay equalized needs to be wide enough to pass the received data signal. The baseband width of a data signal can range from one-half the baud rate up to the baud rate depending on the type of filtering performed. A data signal limited to one-half the baud rate (i.e.: **4800** Hz. for a **9600** baud signal) requires a great deal of precision in the filters, and much control of all the filters, and is not usually practical. On the other hand when the bandwidth is equal to the baud rate (i.e.: 9600 Hz. for a 9600 baud signal) many properties of the signal are easy to control. The IF filter width needs to be twice the baseband width, so the required width would be 9600 Hz to 19200 Hz for a 9600 baud signal (for linear modulation, QAM and PSK), with **19200** Hz resulting in easier to realize modems. If FSK (non-linear modulation) of sufficiently narrow deviation is utilized, the filter width needs to also include the deviation of the FSK in addition to the data spectrum.

Required Group Delay Flatness

The group delay should be flat, within about 20% of a bit duration over the frequency range of interest. Thus, for a 9600 baud signal, where the bit interval is 104 microseconds, the group delay should be flat to within 20 microseconds for best performance, from DC up to 9600 Hz. in the baseband.

Excel is a trademark of Microsoft, Inc.

References

[1] Proakis, John G.,“Digital Communications”, 3rd edition, McGraw-Hill, 1995

[2] McDermott, T.C., “Modem Channel Response”, Proceedings of the TAPR 1995 annual meeting.